How to Solve: Inequality Basics
By BrushMyQuant

YouTube Video Link to this Post is Here

Following is covered in this post

- What is Inequality and Types of Inequalities
- Graphing Inequalities
- Properties of Inequalities
- Types of Inequality Problems
  - x*y > 0
  - x/y > 0
  - x*y < 0
  - x/y < 0
- Basic Problems on Inequalities

What is Inequality and Types of Inequalities

Usually we are given discrete values of variables like x=2, y=3 etc. In case of inequalities we are given a range of values. Let's take some examples to understand this:

x > 3 => x can take all real values which are greater than 3, i.e. 3.001, 4, 5, 6, 8, 100, etc...
So, instead of giving a single value in case of inequalities we are given a set of values for the variables.

Let's understand various types of inequalities now:

Greater Than Inequality (>) : Ex: x > 3 (We have seen above)
Less Than Inequality (<) : Ex: y < 2 => Y can take all read values which are less than 2
Greater Than or Equal to Inequality (≥): Ex: x ≥ 5 (x can take all real values greater than or equal to 5)
Less Than or Equal to Inequality (≤): Ex: y ≤ 3 (y can take all real values less than or equal to 3)
In-Between Inequality (-2 ≤ x < 5): Ex: x can take all values which are greater than or equal to -2 and less than 5

Graphing Inequalities

Now let's talk about how to plot and inequality on the number line

Ex 1: Graph a ≤ 2 and b > -3 on a number line
Sol:
To plot a ≤ 2 we need to draw a line starting at 2 and extending till -∞ on the left hand side (Refer Orange line in below image). Note that we need to darken point 2 because it is included (as a ≤ 2, so 2 is included)
To plot b > -3 we need to draw a line starting at -3 and extending till +∞ on the right hand side. (Refer Green line in below image). Note that we DO NOT darken point -3 as it is excluded (As b > -3 and not ≥ -3)
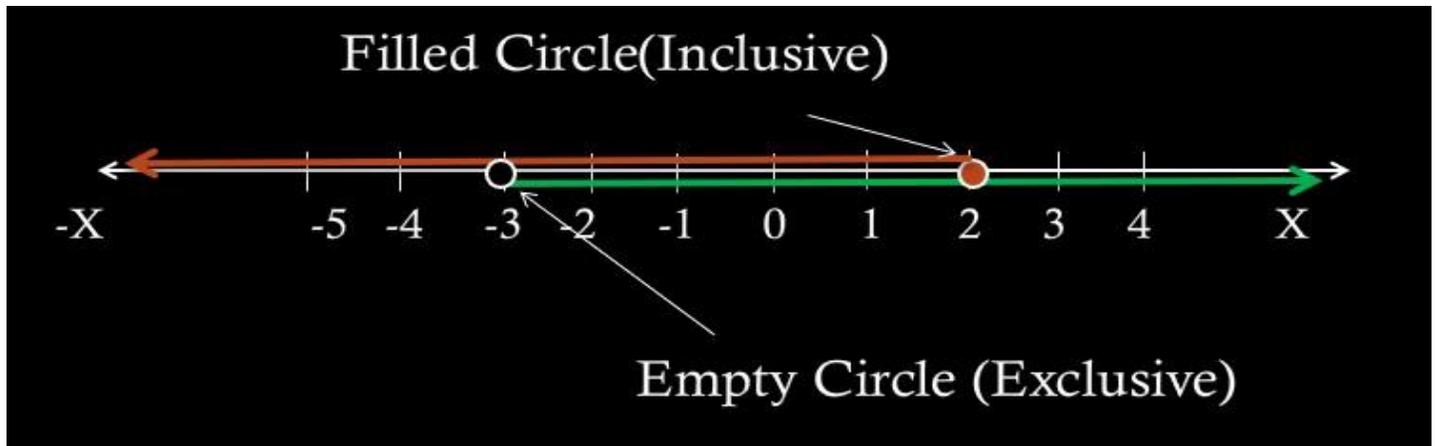
Properties of Inequalities

PROP 1: Adding or Subtracting the same number from both the sides of the inequality DOES NOT change the sign of the inequality.

Ex 1: 7 > 3
Add 4 on both the sides we get
7 + 4 > 3 + 4 => 11 > 7 [Which is True and Note that sign of inequality which was > is still >]

Ex 2: 8 > 4
Subtract 9 from both the sides we get
8 -9 > 4 -9 => -1 > -5 [Which is True and Note that sign of inequality which was < is still <]

Ex 3: a > b
Add k on both the sides we get
a + k > b + k [which is true and sign of inequality did not change]

PROP 2: Multiplying / Dividing an inequality equation with a positive number DOES NOT change the sign of the inequality

Ex 1: 7 > 3
Multiply both the sides by +2 we get
7 * 2 > 3 * 2 => 14 > 6 [which is true and sign of inequality did not change]

Ex 2: 8 > 4
Divide both the sides by +2 we get
$\frac{8}{2} > \frac{4}{2}$ => 4 > 2 [which is true and sign of inequality did not change]

Ex 3: a > b
Multiply both the sides with a positive variable k we get
ak > bk

PROP 3: Multiplying / Dividing an inequality equation with a negative number REVERSES the sign of the inequality

Ex 1: 7 > 3
Multiply both the sides by -2 we get
7 * -2 < 3 * -2 => -14 < -6 [note the sign of inequality has changed from > to <]

Ex 2: 8 > 4
Divide both the sides by -2 we get
$\frac{8}{-2} < \frac{4}{-2}$ => -4 < -2 [note the sign of inequality has changed from > to <]

Ex 3: a > b
Multiply both the sides with a negative variable t we get
at < bt [note the sign of inequality has changed from > to <]

PROP 4: We can add two inequalities which have the same inequality sign

Ex 1:
7 > 3 and
8 > 2, Since the two inequalities have same sign of (>) so we can add both of them to get
7 + 8 > 3 + 2 => 15 > 5

Ex 2:
a > b and
c > d
Since the two inequalities have same sign of (>) so we can add both of them to get
a + c > b + d [Note that this is true irrespective of the signs of a, b, c and d]

Ex 3:
If two inequalities have different signs then we can multiply one of them to make the signs same and then add them
a > b
c < d
we can multiple c < d with -1 to get
-c > -d and now we can add a > b and -c > -d to get
a - c > b - d

PROP 5: Taking Square Root on both sides of an inequality DOES NOT Change the sign of the inequality (provided it is possible to take square root on both the sides and get real values).

Ex 1: $a^2 > b^2$ [given that a and b are positive numbers]
Taking square root on both the sides we will get
a > b

PROP 6: Square of a number is always non-negative

Ex 1: $a^2 \geq 0$ [this is true for all real values of a]
this will be equal to 0 only when a itself is zero

Types of Inequality Problems

Type 1: x * y > 0
If product of two variables > 0 that means that the two variables have SAME SIGN
Either Both are Positive => x > 0 and y > 0
Or Both are Negative => x < 0 and y < 0

Type 2: x / y > 0
If division of two variables > 0 that means that the two variables have SAME SIGN
Either Both are Positive => x > 0 and y > 0
Or Both are Negative => x < 0 and y < 0

Type 3: x * y < 0
If product of two variables < 0 that means that the two variables have DIFFERENT SIGN
Either x > 0 and y < 0
Or x < 0 and y > 0

Type 4: x / y < 0
If division of two variables < 0 that means that the two variables have DIFFERENT SIGN
Either x > 0 and y < 0
Or x < 0 and y > 0

Basic Problems on Inequalities

Q1. Is bd > 0 ?
A. ab > 0
B. cd > 0

Sol:
Stat A: ab > 0
There are two cases
a>0 and b>0
a<0 and b<0
In both the cases we don't know anything about the sign of d so NOT sufficient

Stat B: cd > 0
There are two cases
c>0 and d>0
c<0 and d<0
In both the cases we don't know anything about the sign of b so NOT Sufficient

Combining both the statements we will have four cases
(1) a>0 b>0 c> 0 d>0 (2) a>0 b>0 c< 0 d<0
(3) a<0 b<0 c>0 d>0 (4) a<0 b<0 c<0 d<0
In case 1 and 4 bd > 0 and in case 2 and 3 bd < 0
So, Together also NOT sufficient. So, Answer will be E

Q2. Is bd>0 ?
A. ab > 0
B. ad > 0

Sol:
Stat A: ab > 0
There are two cases
a>0 and b>0
a<0 and b<0
In both the cases we don't know anything about the sign of d so NOT sufficient

Stat B: ad > 0
There are two cases
a>0 and d>0
a<0 and d<0
In both the cases we don't know anything about the sign of b so NOT Sufficient

Combining both the statements we will have two cases
(Since we have a common variable "a" in both the statements so we will combine the two statements based on the sign of the common variable
First case of STAT A will be combined with the first case of Stat B and

Second case of STAT A will be combined with the second case of Stat B

(1) a>0 b>0 d>0 (2) a<0 b<0 d<0

In both the cases bd > 0

So, Together the two statements are sufficient. So, Answer will be C

Q3. Given that y = 4 + (1−x)2(1−x)2. Find yMin(Minimum Value of y) and find the value of x for which y = yMin

Sol:

y = 4 + (1−x)2(1−x)2

We know that square of a number can never be negative, so Min value of (1−x)2(1−x)2 = 0 when 1-x = 0 or when x = 1

So, yMin = 4 when x = 1

How to Solve: Inequality Problems
Algebra and Sine Wave/Wavy Method
By BrushMyQuant

YouTube Video Link to this Post is Here

Following is covered in this post

- **Combining Inequalities**
- **Recap of 4 types of Inequality Problems**
- **Solving Linear Inequalities: Method 1: Algebra**
- **Solving Linear Inequalities: Method 2: Sine Wave Method / Wave Method / Wavy method**

Combining Inequalities

We are discussing this because we will use this in solving problems using the algebra method
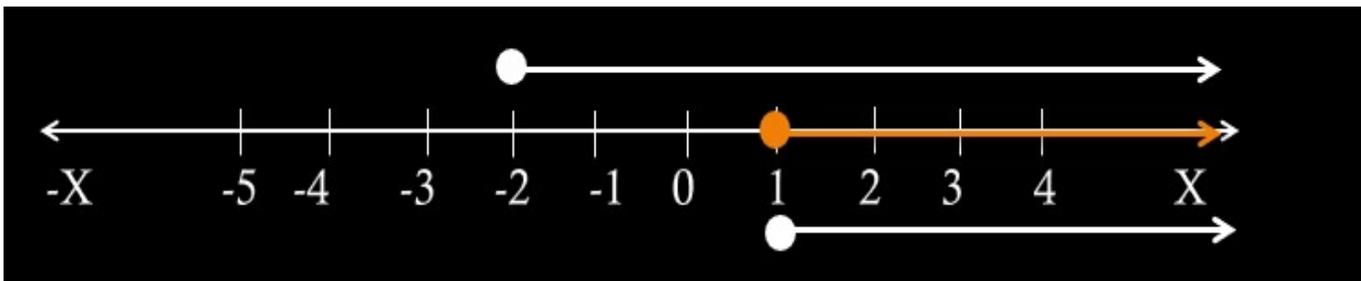
If after solving an inequality equation we are getting x ≥ -2 and x ≥ 1 as two solutions then our final solution will be x ≥ 1
As it is the intersection/common part of both the inequalities
(As shown in orange in above figure)

4 types of Inequality Problems solved using Algebra and Sine Wave Method

There are mainly four types of inequality problems which you would need to solve:--

TYPE 1
x*y > 0
When xy > 0 then we know that both x and y can be either positive or both can be negative
i.e. both x and y have the same sign
so, we have
x>0, y>0 or x<0,y<0

Example Problem
(x-1)*(x-2) > 0

Method 1: Algebra
So, we have two cases
Case 1
both (x-1) and (x-2) are positive
so, x-1 > 0 => x > 1
and x-2 > 0 => x > 2
Intersection of the two cases is x >2

Case 2
both (x-1) and (x-2) are negative
so, x-1 < 0 => x < 1
and x-2 < 0 => x < 2
Intersection of the two cases is x < 1

So, Solution to the question is x < 1 or x > 2

Method 2: Sine Wave Method / Wave Method / Wavy method

In this method we are going to use a sine wave method to solve the problem. Just a quick preview, sine wave is a continuous curve which oscillates between a minimum and a maximum value below and above the base line respectively. Sample image below:
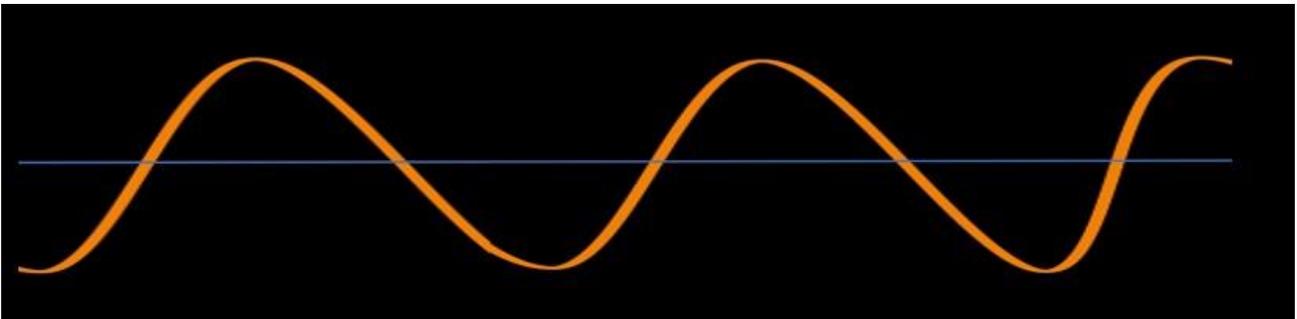
Let's attempt to solve (x-1)*(x-2) > 0 using Sine Wave Method

* Remember that in order to solve the problems using the sine wave method we need to have the coefficient of x positive. [Check out the last part of the video to go through this]

To solve an inequality using this method we find out the intersection points by equating the inequality to 0
=> (x-1)*(x-2) = 0
=> x = 1 or 2

Now, we plot these two points on the number line as shown below
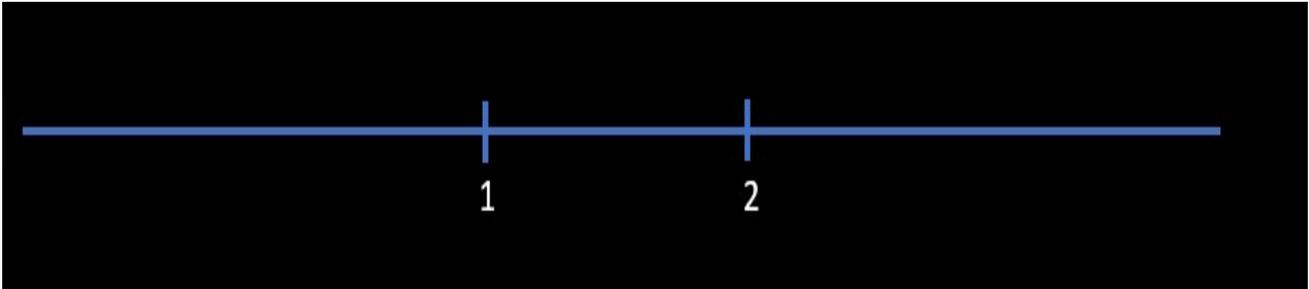

Then we are going to draw a sine curve

- Starting from right top
- Going down at the first solution which is 2 in this case and then
- Coming up in the second solution which is 1 in this case and
- Going down in the third solution if it is there (in this it is not there
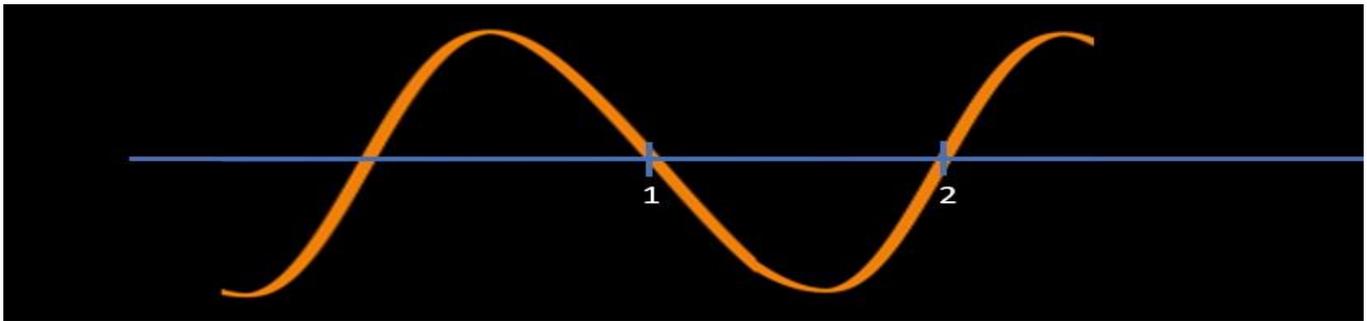
Now we will start marking + and - as mentioned below:
Any Area (in-between) above the number line and below the sine curve is marked as "+" and

Any Area (in-between) below the number line and above the sine curve is marked as "-" as shown below
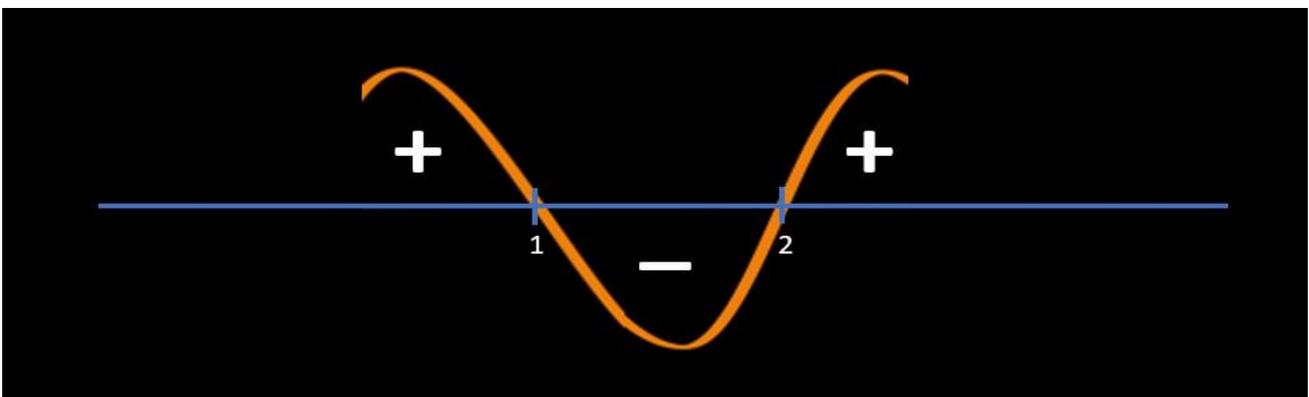
Now, get your answer as below:

- If the inequality in the question is > 0 then pick all the ranges which are "+"
- If the inequality in the question is < 0 then pick all the ranges which are "-"

In our case the question was (x-1)*(x-2) > 0 so we will pick all "+" areas which are
x > 2 and x < 1

If the question was (x-1)*(x-2) < 0 then we will pick all "-" areas which are
1 < x < 2

Note that if the question has ≥ or ≥ then we need to check for the border conditions too
Ex: if question was (x-1)*(x-2) ≥ 0 then we need to check the border condition of x = 1 and x = 2 manually and see if we want to include it in the answer or not.

TYPE 2
x/y > 0
When x/y > 0 then we know that both x and y can be either positive or both can be negative
i.e. both x and y have the same sign
so, we have
x>0, y>0 or x<0,y<0

Example Problem
(x−3)(x−4) > 0

Method 1: Algebra
So, we have two cases
Case 1
Both (x-3) and (x-4) are positive
=> x-3 > 0 => x>3
And x-4 > 0 => x>4
Intersection of the two cases is x > 4

Case 2
Both (x-3) and (x-4) are negative
=> x-3 < 0 => x < 3
and x-4 < 0 => x < 4
Intersection of the two cases is x < 3

So, solution to the question is x < 3 or x > 4

## Method 2: Sine Wave Method / Wave Method / Wavy method

Point of intersections:
x - 3 = 0 and x-4 = 0
=> x = 3, 4

Refer below image
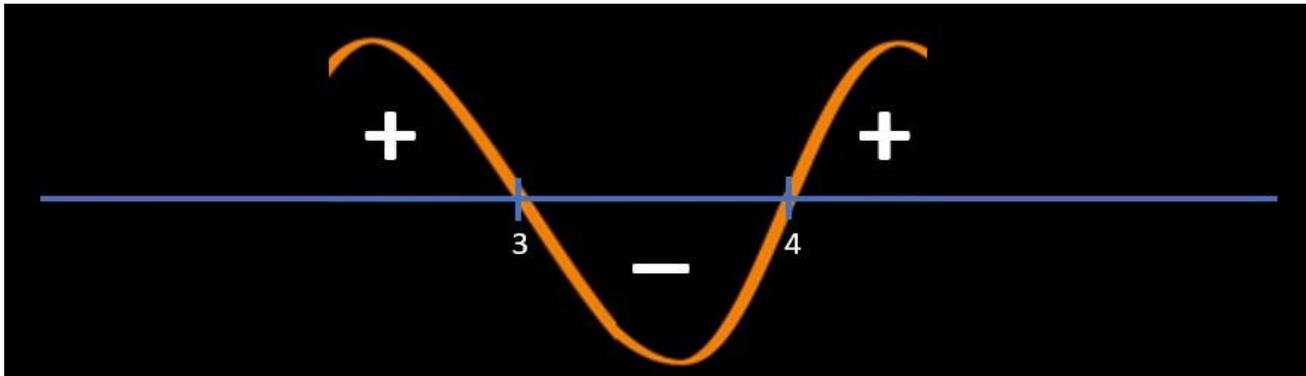
Since question is (x−3)(x−4) > 0
So, we will pick "+" area regions
So, answer is x < 3 and x > 4

TYPE 3
x*y < 0
When x*y < 0 then we know that that
(x can be positive and y will be negative) or (x can be negative and y will be positive)
i.e. x and y have opposite signs
so, we have
x>0, y<0 or x<0,y>0

Example Problem
(x+1)(x-1) < 0

Method 1: Algebra
So, we will have two cases
Case 1
(x+1) is positive and (x-1) is negative
=> x + 1 > 0 => x > -1
And x - 1 < 0 => x < 1
Intersection of the two cases is
-1 < x < 1

Case 2
(x+1) is negative and (x-1) is positive
=> x+1 < 0 => x < -1
And x-1 > 0 => x > 1
The two cases have no intersection. So, no solution from this case

So, solution of the problem is -1 < x < 1

Method 2: Sine Wave Method / Wave Method / Wavy method

Point of intersections:
x + 1 = 0 and x - 1 = 0
=> x = -1, 1

Refer below image
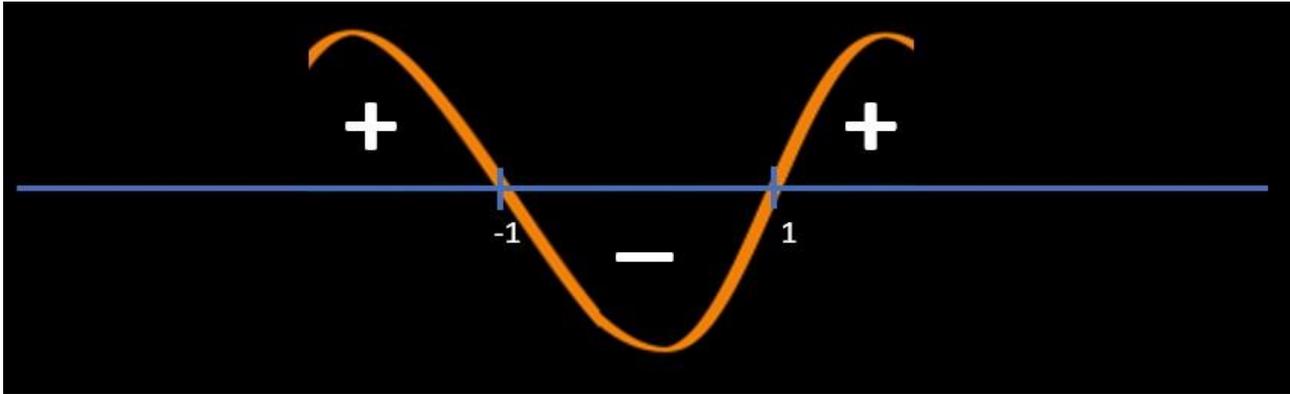
Since question is (x+1)(x-1) < 0
So, we will pick "-" area regions
So, answer is -1 < x < 1

TYPE 4
x/y < 0
When x/y < 0 then we know that that
(x can be positive and y will be negative) or (x can be negative and y will be positive)
i.e. x and y have opposite signs
so, we have
x>0, y<0 or x<0,y>0

Example Problem
(x−2)(x+3) < 0

Method 1: Algebra
So, we will have two cases
Case 1
(x-2) is positive and (x+3) is negative
=> x-2 > 0 => x > 2
And x+3 < 0 or x < -3
There is no intersection of the two cases. So, no solution from this case

Case 2
(x-2) is negative and (x+3) is positive
=> x-2 < 0 => x < 2
And x+3 > 0 => x > -3
Intersection of the two cases is -3 < x < 2

So, Solution of the question is -3 < x < 2

**Method 2: Sine Wave Method / Wave Method / Wavy method**

Point of intersections:
x - 2 = 0 and x + 3 = 0
=> x = 2, -3

Refer below image
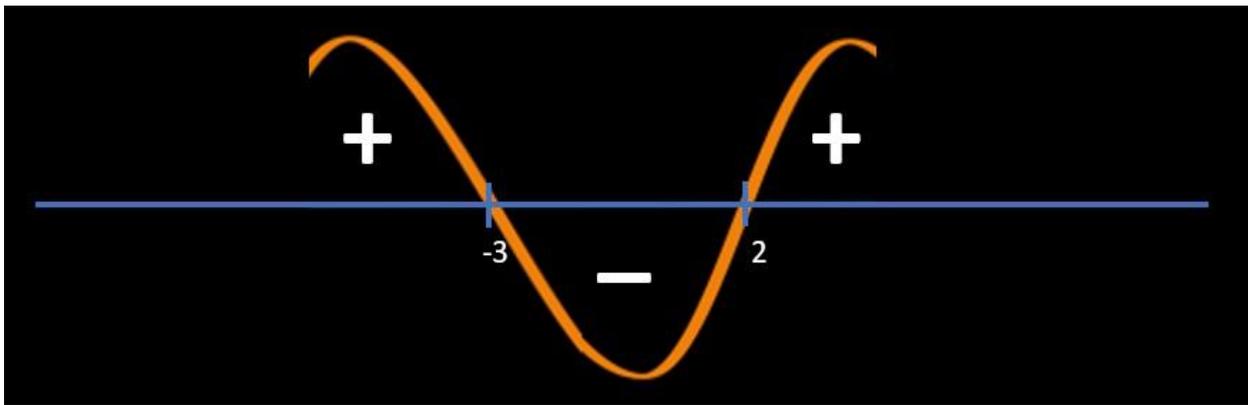
Since question is (x−2)(x+3)(x−2)(x+3) < 0
So, we will pick "-" area regions
So, answer is -3 < x < 2

Sample Problems

1.
If $x^5 + x^2 < 0$, then which one of the following must be true?

(A) $x < -1$
(B) $x < 0$
(C) $x > 0$
(D) $x > 1$
(E) $x^4 < x^2$

Solution:
$x^5 + x^2 < 0$
Taking x^2 common on the left hand side we have
$x^2(x^3+1) < 0$
Now, we know that $x^2(x^3+1) < 0$ that means that product of two terms x^2 and (x^3+1) is < 0 so one of them is positive and other is negative
We also know that x^2 is a square of a number so it can never be $< 0 => (x^3+1) < 0$
or $x^3 < -1$
That means x is a negative number less than 1 as $x^3 = -1$ when x = -1 and for any value of x >-1 $x^3 > -1$
So, Answer will be A

2. Which of the following describes all the values of y for which $y < y^2$?

A. $1 < y$
B. $-1 < y < 0$
C. $y < -1$
D. $1/y < 1$
E. $0 < y < 1$

Solution:
The question can be written as
$y^2 - y > 0$
$=> y*(y-1) > 0$
It is of the form $xy > 0$
So, we will have two cases
Case 1
Both y and y-1 are positive
$=> y > 0$
And $y-1 > 0 => y > 1$
Intersection of the two cases is $y > 1$

Case 2
Both y and y-1 are negative
$=>y < 0$
And $y -1 < 0 => y < 1$

Intersection of the two cases is y <0

So, solution to the problem is y < 0 or y > 1

So, Answer will be D
(As option D can be written as
$1/y - 1 < 0$
or, $(1-y)/y < 0$
or $(y-1)/y > 0$
And solution to this will be same as that of $y*(y-1) > 0$

3. Which of the following describes all values of x for which $1-x^2 >= 0$?

(A) x >= 1
(B) x <= −1
(C) 0 <= x <= 1
(D) x <= −1 or x >= 1
(E) −1 <= x <= 1

Solution:
Question can be written as
$x^2 - 1 <=0$
=> $(x+1)*(x-1) <=0$
Case 1
x+1 is positive or 0 and x-1 is negative or 0
=> x+1 >= 0 => x >= -1
And x-1 <= 0 => x <= 1
Intersection is -1 <= x <= 1

Case 2
x+1 is negative or 0 and x-1 is positive or 0
x+1 <=0 => x <= -1
And x-1 >= 0 => x >= 1
No intersection in this case

So, solution to the problem is -1 <= x <= 1
So, Answer will be E

4. If y>0>x, and $(3+5y)/(x−1) < −7$, then which of the following must be true?

A. 5y−7x+4 < 0
B. 5y+7x−4 > 0
C. 7x−5y−4 < 0
D. 4+5y+7x > 0
E. 7x−5y+4 > 0

Solution:

(3+5y)/(x−1) < −7

=> (3+5y)/(x−1) + 7 < 0

=> ((3+5y) + 7*(x-1))/ (x-1) < 0

=> (7x + 5y -4)/ (x-1) < 0

Now, we know that x < 0 so, x- 1 < 0

in (7x + 5y -4)/ (x-1) < 0

we know that x - 1 < 0

=> (7x + 5y -4) > 0

So, Answer will be B

5. Is $k^2 + k - 2 > 0$?

(1) k < 1

(2) k < -2

Solution:

$k^2 + k - 2 > 0$

=> (k+2)*(k-1) > 0

So, we will have two cases

Case 1

Both k+2 and k -1 positive

k+2 > 0 and k-1 > 0

=> k > -2 and k > 1

Intersection is k > 1

Case 2

Both k+2 and k-1 negative

k+2 < 0 and k -1 < 0

=> k < - 2 and k < 1

intersection is k < -2

So, Solution to the problem is k> 1 or k < -2

So, STAT1 is not SUFFICIENT

STAT2 is SUFFICIENT

So, Answer will be B