How to Solve: Absolute Value Basics

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YouTube Video Link to this Post is Here

Following is Covered in this post

Theory

- **What is Absolute Value / Modulus of a number**
- **Absolute Value on Number Line**
- **Properties of Absolute Values**
- **Absolute Value on Number Line Examples**

After this post please go through Absolute Value Problems and Absolute Value + Inequality post

What is Absolute Value / Modulus of a number

• Absolute Value or modulus (|x|) of a real number x is the non-negative value of the number (x), without any consideration to its sign

Ex

|12| = 12

|-12| = 12 (we just the value after ignoring the sign)

• |x| = x for x >0

= -x for x <0

= 0 for x = 0

Q1. Find the value of |-3| + | 2*3 – 4*2| + |25|

Q2. Find the value of | x+y| where x + z = 20 and y – z = -25

Sol1: 3 + | 6-8 | + 25 = 3 + 2 + 25 = 30

Sol2: x + z = 20 and y – z = -25

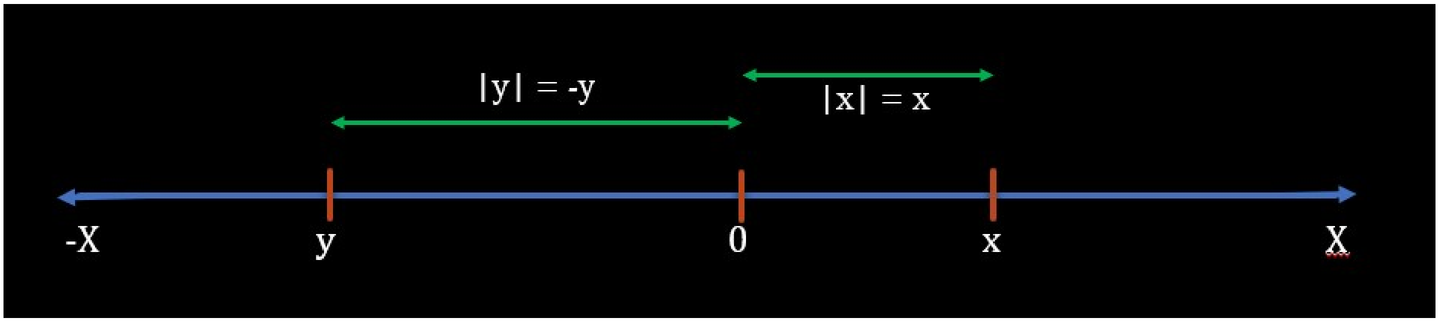
Adding both of them we get x + y = -5

=> | x+y | = |-5| = 5

Absolute Value on Number Line

• **Absolute value of a number x can also be imagined as the distance of that number x from 0 on a number line**

Let's say we have two numbers x and y and x is positive and y is negative. What we are saying is
$|x| = x$ = distance of x from origin
$|y| = -y$ = distance of y from origin
As, shown in the image below:



Properties of Absolute Values

• **PROP 1: Absolute value of a number is always Non-negative**
$|a| \geq 0$ for all values of a
Ex: $|3| = 3 \geq 0$
$|-7| = 7 \geq 0$

• **PROP 2: Minimum value of |a| = 0, when a=0**
Ex: If $|x| = 0 \Rightarrow x = 0$

• **PROP 3: Square root of a number is always positive**
$\sqrt{a^2} = |a|$
Ex:
If $x = \sqrt{25} \Rightarrow x = +5$
But if $x^2 = 25 \Rightarrow x = \pm\sqrt{25} \Rightarrow x = \pm 5$

• **PROP 4: Absolute value of negative of a number is same as absolute value of the number**
$|-a| = |a|$
A derivative of this is
$| a-b | = | b-a |$ because $| b-a | = | -(a-b) |$

• **PROP 5: Product of absolute value of two numbers is same as product of their absolute values**
$|ab| = |a|*|b|$
Ex:
$|7*3| = |7| * |3| = 21$

• **PROP 6: Division of absolute value of two numbers is same as division of their absolute values**

$|ab| = |a|/|b|$

Ex:

$|4/2| = |4|/|2| = 2$

• **PROP 7: Sum of absolute value of two numbers is always ≥ absolute value of their sum**

$|a| + |b| \geq |a+b|$

Ex:

$|7| + |3| \geq |7+3| \Rightarrow 10 \geq 10$

$|5| + |-8| \geq |5 + (-8)| \Rightarrow 13 \geq 3$

• **PROP 8: Difference of absolute value of two numbers is always ≤ absolute value of their difference**

$|a| - |b| \leq |a-b|$

Ex:

$|7| - |3| \leq |7-3| \Rightarrow 4 \leq 4$

$|5| - |-8| \leq |5 - (-8)| \Rightarrow -3 \leq 13$

• **PROP 9: Taking absolute value multiple times or taking it once gives the same result**

$||a|| = |a|$

Ex:

$||-4|| = |-4| \Rightarrow |4| = |-4| = 4$

• **PROP 10: If absolute value of difference of two numbers is zero => both numbers are equal**

$|a-b|=0 \Rightarrow a=b$

Ex:

$| x-4 | =0 > x=4$

Next two will be used a lot to solve absolute values problem!

• **PROP 11: If $|a| \leq b \Rightarrow -b \leq a \leq b$**

• **PROP 12: If $|a| \geq b \Rightarrow a \leq -b$ or $a \geq b$**

• **PROP 13: $|a^n| = |a|^n$**

Ex:

$|-2^4| = |-2|^4 = 16$

• **PROP 14: $|a-b| \geq ||a|-|b||$**

Ex:

$|7-4| \geq ||7|-|4|| \Rightarrow 3 \geq 3$

$|8-(-2)| \geq ||8|-|-2|| \Rightarrow 10 \geq 6$

Q1. If $|a-3| \leq 9$ then find the range of values of a.

Q2. If $|b+5| \geq 10$ then find the range of values of b.

Sol1: $-6 \leq a \leq 12$

Check Video For solution

Sol2: $b \leq -15$ or $b \geq 5$
Check Video For solution

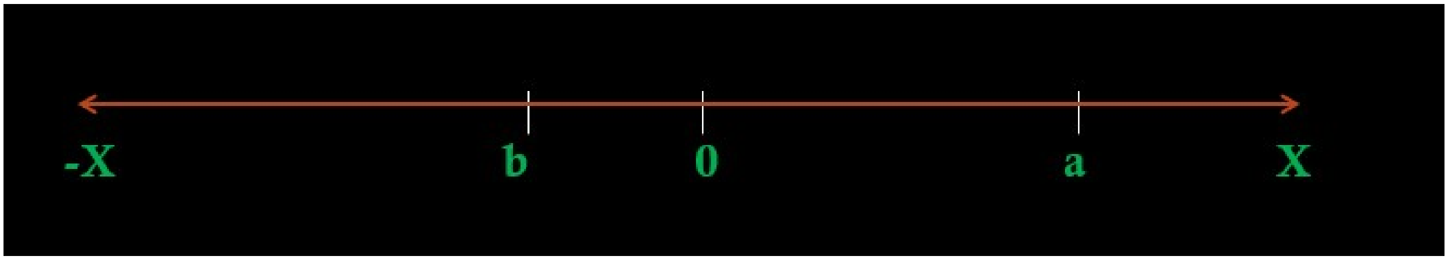
Absolute Value on Number Line Example

If a and b are two variables given then:
|a-b| always means the distance between points a and b
|a+b| = |a| + |b| when a and b have the same sign and
|a+b| = |b| - |a| when a and b have different sign and |b| > |a|

Case 1: a and b are positive and a > b

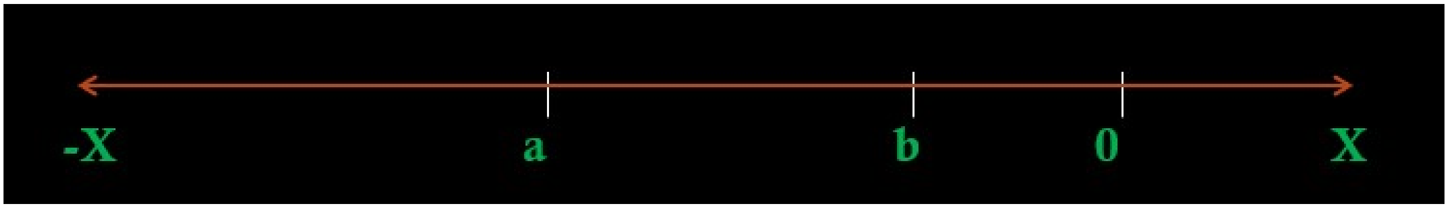
|a-b | = |a| - |b|
|a+b| = |a| + |b|

Case 2: a is positive and b is negative



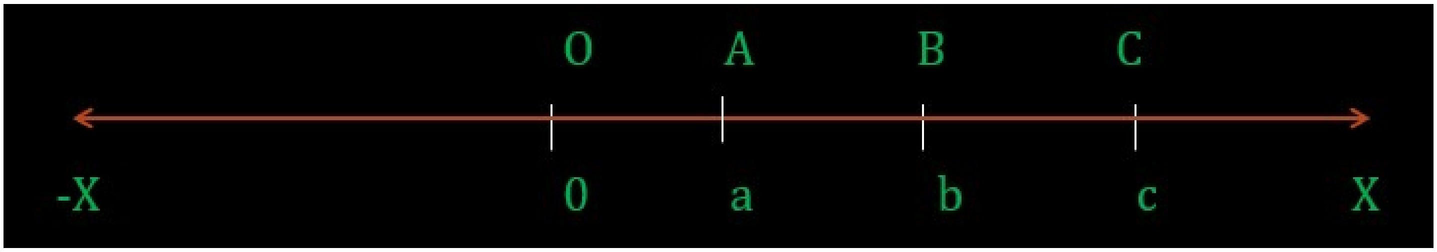
Given: |a| > |b|
|a-b| = |a| + |b|
|a+b| = |a| - |b|

Case 3: Both a and b are negative



Given: |a| > |b|
|a-b| = |a| - |b|
|a+b| = |a| + |b|

Q1. Given the information (below), Simplify |b-a| + |c-b|



Sol:
Method 1
|b-a| = Distance between a and b = AB
|c-b| = Distance between c and b = BC
=> |b-a| + |c-b| = AB = BC = AC = |c-a|

Method 2
|b-a| = |b| - |a|
|c-b| = |c| - |b|
=> |b-a| + |c-b| = |b| - |a| + |c| - |b| = |c| - |a| = |c-a|