

How to Solve: Absolute Value + Inequality Problems

By [BrushMyQuant](#)



YouTube Video Link to this Post is [Here](#)

Before reading this post read [Absolute Value Basics](#) and [Absolute Value Problems](#)

Following is Covered in this post

Theory

- How to open $|x|$
- Basic Property of Absolute value + Inequality
- Problems with one Absolute Value (+ Inequality)
 - Substitution
 - Algebra
- Problems with two Absolute Values (+ Inequality)
 - Substitution
 - Algebra

How to open $|x|$

$$\begin{aligned}|x| &= x \text{ for } x > 0 \\ &= -x \text{ for } x < 0 \\ &= 0 \text{ for } x = 0\end{aligned}$$

Basic Property of Absolute value + Inequality

- If $|x| \leq a \Rightarrow -a \leq x \leq a$
- If $|x| \geq a \Rightarrow x \leq -a \text{ or } x \geq a$

Problems with one Absolute Value (+Inequality): Substitution

• In this method we are going to take smart numbers, numbers which will help us eliminate one or more option choices together, to solve the problem.

Q1. Find all possible range of values of x which satisfy $|x-3| < 3x - 5$?

- A. $x \geq 3$ and $x \leq 3$
- B. $x \geq 3$ and $x < 2$
- C. $x > 3$ and $x < 2$
- D. $x \geq 3$
- E. $x > 2$

Sol1:

We see that Option B and Option C have $x < 2$ and A has $x \leq 3$ So, if we are able to prove that for $x=0$ the equation is not satisfied then we can eliminate all three options together.

$$|x-3| < 3x - 5 \Rightarrow |0-3| < 3*0 - 5 \Rightarrow 3 < -5$$

Which is not true, so A, B, C are eliminated

For D and E let's pick an option choice between 2 and 3 let's say 2.5 and if it satisfies

$$|x-3| < 3x - 5 \Rightarrow |2.5-3| < 3*2.5 - 5 \Rightarrow 0.5 < 2.5$$

which is true, so D is the answer

Q2. Find all possible range of values of x which satisfy $|3x-1| \geq 2x + 4$?

- A. $x \geq 1/3$ and $x \leq 1/3$
- B. $x \geq 1/3$ and $x < -3/5$
- C. $x > 5$ and $x \leq 1/3$
- D. $x \geq 5$ and $x \leq -3/5$
- E. $x > 5$ and $x < -3/5$

Sol2:

In this case we can see that A and B have $x \geq 1/3$ and C,D,E have $x > 5$, so let's try a value of x between 1/3 and 5 (Say $x=1$) and see if it satisfies

$$|3x-1| \geq 2x + 4 \Rightarrow |3*1-1| \geq 2*1 + 4 \Rightarrow 2 \geq 6 \text{ which is false}$$

So, A and B are eliminated

For C, D and E we see that D has $x \geq 5$ and C and E have $x > 5$. So, let's try $x=5$ and see

$$|3x-1| \geq 2x + 4 \Rightarrow |3*5-1| \geq 2*5 + 4 \Rightarrow 14 \geq 14 \text{ which is true}$$

So, D is the answer

Problems with one Absolute Value (+Inequality) : Algebra

• In this method we are going to consider two cases, in one case we will assume that the value inside the absolute value is ≥ 0 and in the second case we will assume that the value inside the absolute value is < 0 and solve the two cases.

Q1. Find all possible range of values of x which satisfy $|x-3| < 3x - 5$?

- A. $x \geq 3$ and $x \leq 3$
- B. $x \geq 3$ and $x < 2$
- C. $x > 3$ and $x < 2$
- D. $x \geq 3$
- E. $x > 2$

Sol1:

Case 1

$$x - 3 \geq 0 \Rightarrow x \geq 3$$

$$\Rightarrow |x-3| = x-3$$

$$\Rightarrow x-3 < 3x - 5$$

$$\Rightarrow x > 1$$

But one condition was $x \geq 3$. So, our final answer will be the intersection of $x > 1$ and $x \geq 3$ which is $x \geq 3$
[Check out the link for Inequalities in my signature to understand this part in bit more detail]

Case 2

$$x - 3 < 0 \Rightarrow x < 3$$

$$\Rightarrow |x-3| = -(x-3)$$

$$\Rightarrow -(x-3) < 3x - 5$$

$$\Rightarrow 4x > 8$$

$$\Rightarrow x > 2$$

But our condition was $x < 3$

So, Answer is $x \geq 3$

Q2. Find all possible range of values of x which satisfy $|3x-1| \geq 2x + 4$?

- A. $x \geq 1/3$ and $x \leq 1/3$
- B. $x \geq 1/3$ and $x < -3/5$
- C. $x > 5$ and $x \leq 1/3$
- D. $x \geq 5$ and $x \leq -3/5$
- E. $x > 5$ and $x < -3/5$

Sol2:

Case 1

$$3x-1 \geq 0 \Rightarrow x \geq 1/3$$

$$\Rightarrow |3x-1| = 3x-1$$

$$\Rightarrow 3x-1 \geq 2x + 4$$

$$\Rightarrow x \geq 5$$

Which is inside the range so $x \geq 5$ is a solution

Case 2

$$3x-1 < 0 \Rightarrow x < 1/3$$

$$\Rightarrow |3x-1| = -(3x-1)$$

$$\Rightarrow -(3x-1) \geq 2x + 4$$

$$\Rightarrow 5x \leq -3$$

$$\Rightarrow x \leq -3/5$$

Which is inside the range so $x \leq -3/5$ is a solution

So, D is the answer

Problems with two Absolute Value (+Inequality) : Substitution

• In this method we are going to take smart numbers, numbers which will help us eliminate one or more option choices together, to solve the problem.

Q1. Find all possible range of values of x which satisfy $|x+1| + |x+2| > 3x+1$?

A. $x \geq 2$ and $x \leq -2$

B. $x \geq -1$ and $x < -2$

C. $-2 \leq x < 2$

D. $x < 2$

E. $-2 \leq x < -1$

Sol1:

For A and B we will pick a value of $x > 2$ (let's say $x=3$) to eliminate both of these option choices.

$$|x+1| + |x+2| > 3x+1 \Rightarrow |3+1| + |3+2| > 3*3+1 \Rightarrow 9 > 10$$

which is not true \Rightarrow A and B are eliminated

For C, D and E Let's pick a value of x which is < -2 (let's say $x=-10$)

$$\Rightarrow |x+1| + |x+2| > 3x+1 \Rightarrow |-10+1| + |-10+2| > 3*-10+1$$

$$\Rightarrow 17 > -29$$

Which is true \Rightarrow D is the answer

Q2. Find all possible range of values of x which satisfy $|2x+3| + |3x+4| < 6x+5$?

A. $x > 2$ and $x \leq -3/2$

B. $-3/2 \leq x \leq -4/3$

C. $x > 2$ and $-3/2 \leq x \leq -4/3$

D. $x \geq -4/3$

E. $x > 2$

Sol2:

For A, B, C we can substitute $x = -3/2$ and check if it satisfies

$$|2x+3| + |3x+4| < 6x+5 \Rightarrow |2*-3/2 + 3| + |3*-3/2 + 4| < 6*-3/2 + 5$$

which will be false as left hand side will be non-negative and right hand side is negative

So, A, B and C are eliminated

For D and E we can pick a value of $x > 2$ (Say $x=3$) and check if it satisfies

$$|2x+3| + |3x+4| < 6x+5 \Rightarrow |2*3+3| + |3*3+4| < 6*3+5 \Rightarrow 9 + 13 < 23$$

which is true \Rightarrow E is the answer

Problems with two Absolute Value (+Inequality) : Algebra

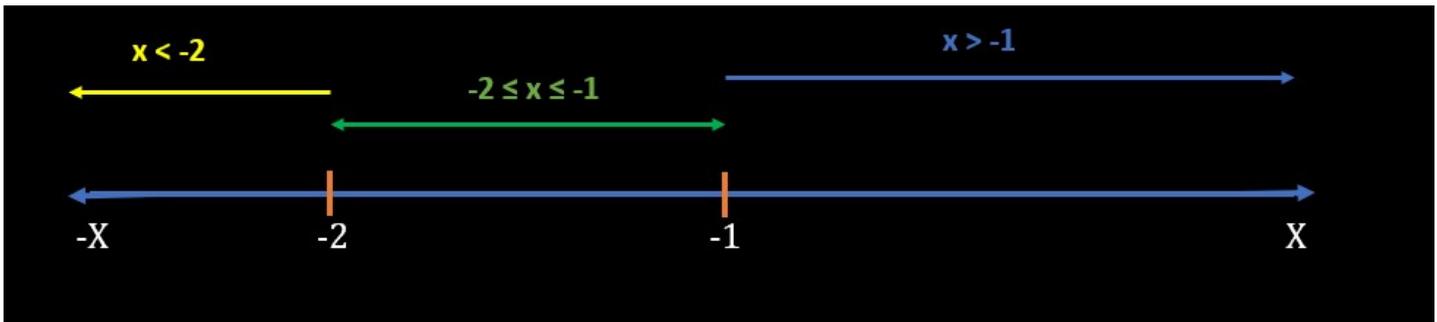
• In this method we are going to assume that the value inside the two absolute value is zero and take down the points. We will plot the points on the number line and divide the number line into three parts and then solve after opening the absolute value in these three cases.

Q1. Find all possible range of values of x which satisfy $|x+1| + |x+2| > 3x+1$?

- A. $x \geq 2$ and $x \leq -2$
- B. $x \geq -1$ and $x < -2$
- C. $-2 \leq x < 2$
- D. $x < 2$
- E. $-2 \leq x < -1$

Sol1:

We will assume $x+1=0$ and $x+2=0$ and get $x=-1$ and $x=-2$ as two points on the number line. We will plot the points on the number line and split the number line into three parts as shown in the image below



Case 1

$$x > -1$$

If $x > -1$ then take any value of x , let's say $x=0$ and check if the value inside the two absolute values is positive or negative

Both $x+1$ and $x+2$ are positive

$$\Rightarrow |x+1| = x+1 \text{ and } |x+2| = x+2$$

$$\Rightarrow x+1 + x+2 > 3x+1$$

$$\Rightarrow x < 2$$

But our condition was $x > -1$

So, our solution will be the intersection of these two which is nothing but $-1 < x < 2$

Case 2

$$-2 \leq x \leq -1$$

If $-2 \leq x \leq -1$ then take any value of x , let's say $x=-1.5$ and check if the value inside the two absolute values is positive or negative

$x+1$ will be negative and $x+2$ will be positive

$$\Rightarrow |x+1| = -(x+1) \text{ and } |x+2| = x+2$$

$$\Rightarrow -x-1 + x+2 > 3x+1$$

$$\Rightarrow x < 0$$

But our condition was $-2 \leq x \leq -1$

So, our solution will be the intersection of these two which is nothing but $-2 \leq x \leq -1$

Case 3

$x < -2$

If $x < -2$ then take any value of x , let's say $x = -3$ and check if the value inside the two absolute values is positive or negative

Both $x+1$ and $x+2$ are negative

$\Rightarrow |x+1| = -(x+1)$ and $|x+2| = -(x+2)$

$\Rightarrow -x-1 - x-2 > 3x+1$

$\Rightarrow x < -4/5$

But our condition was $x < -2$

So, our solution will be $x < -2$

So, our solution is $-1 < x < 2$, $-2 \leq x \leq -1$ and $x < -2$

So, combined solution is $x < 2$

Q2. Find all possible range of values of x which satisfy $|2x+3| + |3x+4| < 6x+5$?

A. $x > 2$ and $x \leq -3/2$

B. $-3/2 \leq x \leq -4/3$

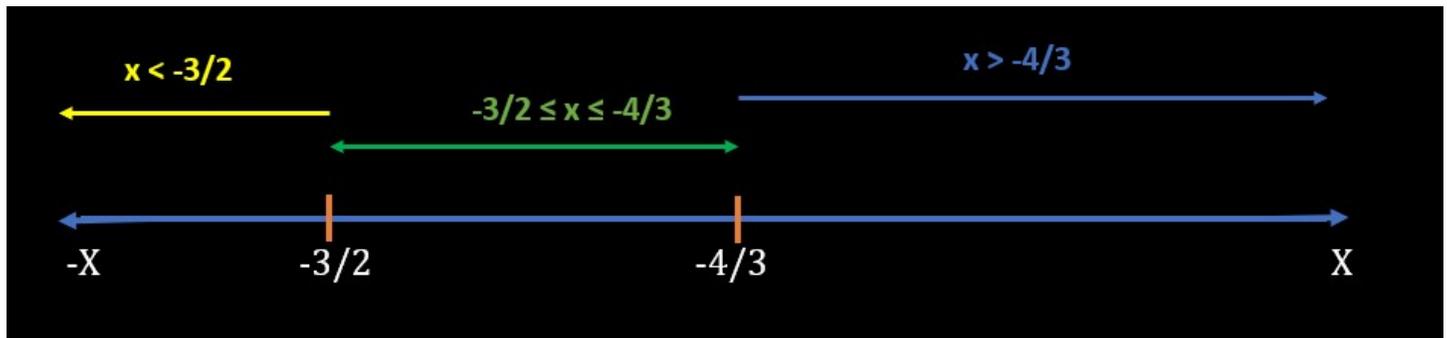
C. $x > 2$ and $-3/2 \leq x \leq -4/3$

D. $x \geq -4/3$

E. $x > 2$

Sol2:

We will assume $2x+3=0$ and $3x+4=0$ and get $x=-3/2$ and $x=-4/3$ as two points on the number line. We will plot the points on the number line and split the number line into three parts as shown in the image below



Case 1

$x > -4/3$

If $x > -4/3$ then take any value of x , let's say $x=0$ and check if the value inside the two absolute values is positive or negative

Both $2x+3$ and $3x+4$ are positive

$\Rightarrow 2x+3 + 3x+4 < 6x+5$

$\Rightarrow x > 2$

Which is in the range, so $x > 2$ is one solution

Case 2

$$-3/2 \leq x \leq -4/3$$

If $-3/2 \leq x \leq -4/3$ then take any value of x , let's say $x=-1.4$ and check if the value inside the two absolute values is positive or negative

$2x+3$ will be positive and $3x+4$ will be negative

$$\Rightarrow 2x+3 - (3x+4) < 6x+5$$

$$\Rightarrow 7x > -6$$

$$\Rightarrow x > -6/7$$

But our condition was $-3/2 \leq x \leq -4/3$

So, No solution here

Case 3

$$x < -3/2$$

If $x < -3/2$ then take any value of x , let's say $x=-10$ and check if the value inside the two absolute values is positive or negative

Both $2x+3$ and $3x+4$ will be negative

$$\Rightarrow -2x-3 - (3x+4) < 6x+5$$

$$\Rightarrow 11x > -12$$

$$\Rightarrow x > -12/11$$

But our condition was $x < -3/2$

So, No solution here

So, Solution is Option E $x > 2$