

How to Solve: Absolute Value Problems

By [BrushMyQuant](#)



YouTube Video Link to this Post is [Here](#)

Before reading this post read [Absolute Value Basics](#) Post and after reading this post please read [Absolute Value + Inequalities](#) Post

Following is Covered in this post

Theory

- Absolute Value of x
- Problems with one Absolute Value
 - Substitution
 - Squaring Both Sides
 - Algebra
- Problems with two Absolute Values
 - Substitution
 - Algebra

Absolute Value of x

$$\begin{aligned} |x| &= x \text{ for } x > 0 \\ &= -x \text{ for } x < 0 \\ &= 0 \text{ for } x = 0 \end{aligned}$$

Problems with one Absolute Value: Substitution

• In this method we are going to substitute the answer choices and check which answer choice satisfies the answer.

Q1. Which value of x satisfies $|x-3| = 3x$?

- A. 0
- B. $-3/2$
- C. $3/4$
- D. 1
- E. 2

Sol1:

- A. $|0-3| = 3*0 \Rightarrow 3 = 0$ FALSE
- B. $|-3/2-3| = 3*(-3/2) \Rightarrow 9/2 = -9/2$ FALSE
- C. $|3/4-3| = 3*3/4 \Rightarrow 9/4 = 9/4$ TRUE, So C is the answer. WE don't need to check further but still showing the work for completing the solution
- D. $|1-3| = 3*1 \Rightarrow 2 = 3$ FALSE
- E. $|2-3| = 3*2 \Rightarrow 1 = 6$ FALSE

Q2. Which value of x satisfies $|x-4| = 5 + 2x$?

- A. 0 and $-1/3$
- B. $-1/3$
- C. 1 and $-1/3$
- D. 2 and $-1/3$
- E. -9 and $-1/3$

Sol2: Since $-1/3$ is in all the option choices so we will not check this value

- A. 0 and $-1/3 \Rightarrow |0-4| = 5 + 2*0 \Rightarrow 4 = 5$ FALSE
- B. $-1/3 \Rightarrow$ Leave for now
- C. 1 and $-1/3 \Rightarrow |1-4| = 5 + 2*1 \Rightarrow 3 = 7$ FALSE
- D. 2 and $-1/3 \Rightarrow |2-4| = 5 + 2*2 \Rightarrow 2 = 9$ FALSE
- E. -9 and $-1/3 \Rightarrow |-9-4| = 5 + 2*(-9) \Rightarrow 13 = -13$ FALSE

So, Answer is B

Problems with one Absolute Value: Squaring Both Sides

• In this case we will be taking absolute value term to one side and everything else to the other side. Then, we will square both the sides to eliminate absolute value and solve the quadratic. We will then substitute the answer choices back in the question and check if the answer satisfies the question or not.

Q1. Which value of x satisfies $|x-3| = 3x$?

- A. 0
- B. $-3/2$
- C. $3/4$
- D. 1
- E. 2

Sol1:

$$|x-3| = 3x$$

Squaring both the sides we have

$$(x-3)^2 = (3x)^2$$

$$x^2 - 6x + 9 = 9x^2$$

$$\Rightarrow 8x^2 + 6x - 9 = 0$$

$$\Rightarrow 8x^2 + 12x - 6x - 9 = 0$$

$$\Rightarrow 4x(2x+3) - 3(2x+3) = 0$$

$$\Rightarrow (4x-3)(2x+3) = 0$$

$$\Rightarrow x = 3/4 \text{ or } -3/2$$

Now, substitute both the values in the question and check which one satisfies. As seen in the case of substitution only $3/4$ will satisfy.

Q2. Which value of x satisfies $|x-4| = 5 + 2x$?

- A. 0 and $-1/3$
- B. $-1/3$
- C. 1 and $-1/3$
- D. 2 and $-1/3$
- E. -9 and $-1/3$

Sol2:

$$|x-4| = 5 + 2x$$

Squaring both the sides we have

$$(x-4)^2 = (5+2x)^2$$

$$\Rightarrow x^2 - 8x + 16 = 25 + 20x + 4x^2$$

$$\Rightarrow 3x^2 + 28x + 9 = 0$$

$$\Rightarrow 3x^2 + x + 27x + 9 = 0$$

$$\Rightarrow x(3x+1) + 9(3x+1) = 0$$

$$\Rightarrow (x+9) * (3x+1) = 0$$

$$\Rightarrow x = -9, -1/3$$

Now, substitute both the values in the question and check which one satisfies. As seen in the case of substitution only $-1/3$ will satisfy.

Problems with one Absolute Value: Algebra

- In this method we are going to use the property of $|x|$

$$|x| = x \text{ for } x \geq 0$$

$$= -x \text{ for } x < 0$$

and we will create two cases and solve both of them individually.

Q1. Which value of x satisfies $|x-3| = 3x$?

- A. 0
- B. $-3/2$
- C. $3/4$
- D. 1
- E. 2

Sol1:

Case 1: Value inside the absolute value is positive

$$x-3 \geq 0 \Rightarrow x \geq 3$$

$$\Rightarrow |x-3| = x-3$$

$$\text{So, } |x-3| = 3x \Rightarrow x-3 = 3x$$

$$\Rightarrow x = -3/2$$

But our condition was $x \geq 3$, so this is NOT a solution

Case 2: Value inside the absolute value is negative

$$x-3 < 0 \Rightarrow x < 3$$

$$\Rightarrow |x-3| = -(x-3)$$

$$\text{So, } |x-3| = 3x \Rightarrow -(x-3) = 3x$$

$$\Rightarrow x = 3/4$$

This is true as our condition was $x < 3$

So, answer is $x = 3/4$

Q2. Which value of x satisfies $|x-4| = 5 + 2x$?

- A. 0 and $-1/3$**
- B. $-1/3$**
- C. 1 and $-1/3$**
- D. 2 and $-1/3$**
- E. -9 and $-1/3$**

Sol2:

Case 1: Value inside the absolute value is positive

$$x-4 \geq 0 \Rightarrow x \geq 4$$

$$\Rightarrow |x-4| = x-4$$

$$\text{So, } |x-4| = 5 + 2x \Rightarrow x-4 = 5 + 2x$$

$$\Rightarrow x = -9$$

But our condition was $x \geq 4$, so this is NOT a solution

Case 2: Value inside the absolute value is negative

$$x-4 < 0 \Rightarrow x < 4$$

$$\Rightarrow |x-4| = -(x-4)$$

$$\text{So, } |x-4| = 5 + 2x \Rightarrow -(x-4) = 5 + 2x$$

$$\Rightarrow x = -1/3$$

This is true as our condition was $x < 4$

So, answer is $x = -1/3$

Problems with two Absolute Values: Substitution

- In this problem we are going to substitute the answer choices and check which value satisfies.

Q1. Which value of x satisfies $|x+1| + |x+2| = 3x$?

- A. 3**
- B. 3 and $-1/3$**
- C. 3 and $-3/5$**
- D. 3 and $1/3$**
- E. 3, $1/3$, $-1/3$ and $-3/5$**

Sol1: A

Check [video](#) for explanation

Q2. Which value of x satisfies $|2x-4| + |3x+6| = 6x$?

- A. 2
- B. 2 and $-2/11$
- C. 2 and $-10/7$
- D. 2, $-2/11$ and $-10/7$

Sol2: A

Check [video](#) for explanation

Problems with two Absolute Values : Algebra

• In this method we are going to assume that the value inside the two absolute value is zero and take down the points. We will plot the points on the number line and divide the number line into three parts and then solve then open the absolute value in these three cases.

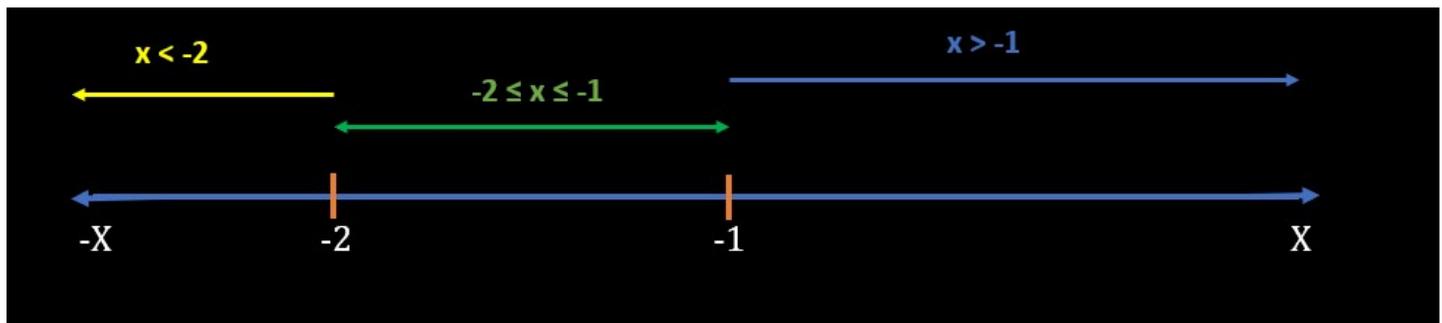
Q1. Which value of x satisfies $|x+1| + |x+2| = 3x$?

- A. 3
- B. 3 and $-1/3$
- C. 3 and $-3/5$
- D. 3 and $1/3$
- E. 3, $1/3$, $-1/3$ and $-3/5$

Sol1:

Assuming $x+1 = 0$ and $x+2 = 0$ we get $x = -1$ and $x = -2$

We will plot these two points on the number line and take three cases as shown in the image below



Case 1

$x > -1$

If $x > -1$ then take any value of x , let's say $x=0$ and check if the value inside the two absolute values is positive or negative

Both $x+1$ and $x+2$ are positive

$\Rightarrow |x+1| = x+1$ and $|x+2| = x+2$

$$\Rightarrow x+1 + x+2 = 3x$$

$$\Rightarrow x = 3$$

Which is true as our condition was $x > -1$

Case 2

$$-2 \leq x \leq -1$$

If $-2 \leq x \leq -1$ then take any value of x , let's say $x=-1.5$ and check if the value inside the two absolute values is positive or negative

$x+1$ will be negative and $x+2$ will be positive

$$\Rightarrow |x+1| = -(x+1) \text{ and } |x+2| = x+2$$

$$\Rightarrow -x-1 + x+2 = 3x$$

$$\Rightarrow x = 1/3$$

But our condition was $-2 \leq x \leq -1$ so it is NOT a solution

Case 3

$$x < -2$$

If $x < -2$ then take any value of x , let's say $x=-3$ and check if the value inside the two absolute values is positive or negative

Both $x+1$ and $x+2$ are negative

$$\Rightarrow |x+1| = -(x+1) \text{ and } |x+2| = -(x+2)$$

$$\Rightarrow -(x+1) + -(x+2) = 3x$$

$$\Rightarrow x = -3/5$$

But our condition was $x < -2$ so it is NOT a solution

So, solution is $x = 3$

Q2. Which value of x satisfies $|2x-4| + |3x+6| = 6x$?

A. 2

B. 2 and $-2/11$

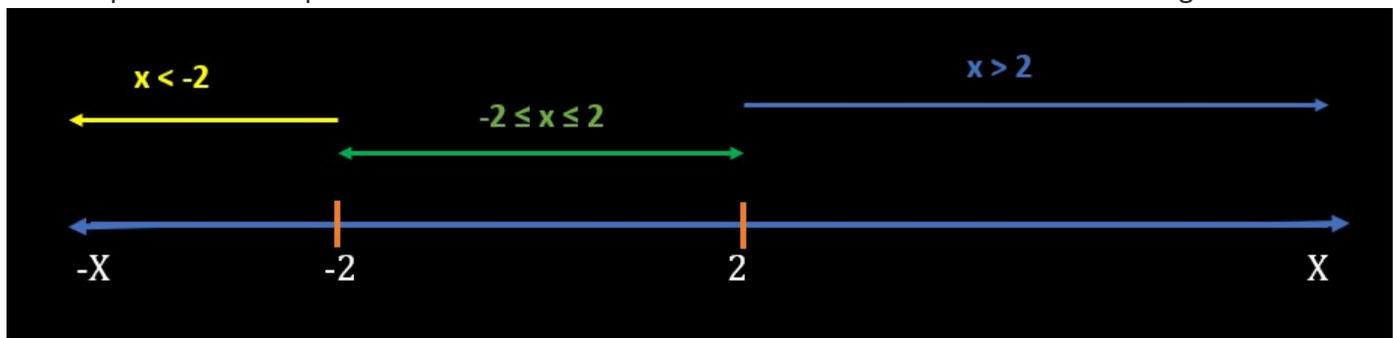
C. 2 and $-10/7$

D. 2, $-2/11$ and $-10/7$

Sol2:

Assuming $2x-4 = 0$ and $3x+6 = 0$ we get $x = 2$ and $x = -2$

We will plot these two points on the number line and take three cases as shown in the image below



Case 1

$$x > 2$$

If $x > 2$ then take any value of x , let's say $x=3$ and check if the value inside the two absolute values is positive or negative

Both $2x-4$ and $3x+6$ are positive

$$\Rightarrow |2x-4| = 2x-4 \text{ and } |3x+6| = 3x+6$$

$$\Rightarrow 2x-4 + 3x+6 = 6x$$

$$\Rightarrow x = 2$$

Now 2 is in the boundary and if you will check then 2 actually satisfies the answer. (If we would have taken $x \geq 2$ then 2 would be in the answer choice, we will get 2 answer in the second case now)

Case 2

$$-2 \leq x \leq 2$$

If $-2 \leq x \leq 2$ then take any value of x , let's say $x=0$ and check if the value inside the two absolute values is positive or negative

$2x-4$ will be negative and $3x+6$ will be positive

$$\Rightarrow |2x-4| = -(2x-4) \text{ and } |3x+6| = 3x+6$$

$$\Rightarrow -2x+4 + 3x+6 = 6x$$

$$\Rightarrow x = 2$$

Which is true as our range was $-2 \leq x \leq 2$

Case 3

$$x < -2$$

If x then take any value of x , let's say $x=-3$ and check if the value inside the two absolute values is positive or negative

Both $2x-4$ and $3x+6$ are negative

$$\Rightarrow |2x-4| = -(2x-4) \text{ and } |3x+6| = -(3x+6)$$

$$\Rightarrow -2x+4 - 3x-6 = 6x$$

$$\Rightarrow x = -2/11$$

But our condition was $x < -2$ so it is NOT a solution

So, solution is $x = 2$