How to Solve: Even and Odd Numbers

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YouTube Video Link to this Post is Here

Following is Covered in this post

Theory of Even and Odd Numbers

- **What are Even Numbers ?**
- **Even Number Problem ?**
- **What are Odd Numbers ?**
- **Odd Numbers Problem ?**
- **Properties of Even and Odd Numbers ?**
- **Solved Problems**

What are Even Numbers

A number which gives 0 as remainder when divided by 2 is an even number.

- Example: 18

- Even numbers end with a units' digit of 0, 2, 4, 6, 8
- An even number "n" is represented as
n = 2k , where k is an integer

- Example: Consecutive even numbers can be taken as
2k-4, 2k-2, 2k, 2k+2, 2k+4

Even Numbers Problem: Sum of three consecutive even numbers is 60. Find the numbers?

Sol: Let the three numbers be 2k-2, 2k, 2k+2
=> 2k-2 + 2k + 2k+2 = 60
=> 6k = 60
=> k = 10
=> Numbers are 18, 20, 22

What are Odd Numbers

A number which gives 1 as remainder when divided by 2 is odd number.

- Example: 19

- Odd numbers end with a units' digit of 1, 3, 5, 7, 9
- An odd number "n" is represented as
n = 2k + 1 or 2k-1 , where k is an integer

- Example: Consecutive odd numbers are
2k-5, 2k-3, 2k-1, 2k+1, 2k+3, 2k+5

Odd Numbers Problem: Sum of 4 consecutive odd numbers is 80. Find the numbers?

Sol: Let the three numbers be 2k-3, 2k-1, 2k+1, 2k+3
=> 2k-3 + 2k-1 + 2k+1 + 2k+3 = 80
=> 8k = 80
=> k = 10
=> Numbers are 17, 19, 21, 23

Properties of Even and Odd Numbers

Addition and Subtraction	
Addition E + E = E E + O = O O + E = O O + O = E Adding Odd number of Odds will give us O Adding even number of Odds will give us E where E -> Even, O -> Odd	**Subtraction** E – E = E E – O = O O – E = O O – O = E Subtracting odd number of Odds will give us O Subtracting even number of Odds will give us E where E -> Even, O -> Odd

Division and Multiplication	
Division E / E = E or F or O E / O = E or F O / E = F O / O = O or F where E -> Even, O -> Odd, F -> Fraction	**Multiplication** E * E = E => $E^{+ve\ Integer}$ = E E * O = E O * E = E O * O = O => $O^{+ve\ Integer}$ = O where E -> Even, O -> Odd

➽ Product of numbers will be even when there is at least one number is Even.

Example: 3*3*2 = 18 = Even as there was one even number 2 on the left side

➽ Product of numbers will be odd ONLY when all numbers are odd.

Example: 3*3*3 = 27 = Odd as all the numbers on the left side were odd

Solved Problems

Q1. If x, y, and z are integers and x + yz is odd, then which of the following must be true?
I. x + z is even
II. x + y is odd
III. y + z is odd
IV. xy is even
V. yz is even
VI. xz is odd
VII. xyz is even

Sol: x + yz = O
=> We will have four cases

Either x is E and yz is O.
=> Only one case possible
Case 1: x = E, y = O, z = O

Or x is O and yz is E
=> Three cases possible
Case 2: x = O, y = E, z = O
Case 3: x = O, y = O, z = E
Case 4: x = O, y = E, z = E

I. x + y is odd
We have to check for x + y in all the 4 possible cases
For x + y to be odd, one has to even and other has to be odd
But in 3rd case both x and y are odd. So, not possible
II. y + z is odd
Similar logic, in case 1 and case 4 its not possible
III. x + z is even
Similar logic, in case 1, 3,4 its not possible
IV. xy is even
At least one has to even. Not possible in case 3
V. yz is even
At least one has to even. Not possible in case 1
VI. xz is odd
Both have to odd. Not possible in case 1,3,4
VII. xyz is even
At least one has to even. Possible in all the cases

Answer VII

Q2.If x is even, y is odd, z is even, then whether the following are odd or even
I. x + yz
II. x + y + yz
III. xy + z
IV. (x+1)*(y+1)*(z+1)
V. xy*(z+1)
VI. $(x+1)^2 * y * (z+1)^3$

Sol:
I. E + O*E = E + E = E
II. E + O + O*E = E + O + E = O
III. E*O + E = E + E = E
IV. (E + O) * (O + O) * (E + O) = O (E * O = E
V. E *O * (E + O) = E
VI. $(E+O)^2 * O * (E+O)^3$ = O * O * O = O

Q3. Product of 4 consecutive numbers will be divisible by all of the following EXCEPT?
A. 6
B. 8
C. 12
D. 24
E. 48?

Sol: Let's take values to solve this. Lets take 4 numbers as 1, 2, 3, 4
Their product = 1*2*3*4 = 24 and will be divisible by all numbers except E
Answer E

<u>Theory</u>: Product of n consecutive numbers will always be divisible by n!

Q4. Sum of three consecutive even numbers is divisible by all of the following EXCEPT
A. 1
B. 2
C. 3
D. 4
E. 6

Sol: Let numbers be 2k-2, 2k, 2k+2
=> Sum = 6k
=> Will be divisible by 6 and all factors of 6
=> Divisible by 1, 2 , 3 and 6
Answer D

Q5. Sum of three consecutive odd numbers is divisible by which of the following
A. 2
B. 3
C. 4
D. 5
E. 6

Sol: Let numbers be 2k+1, 2k+3, 2k+5
=> Sum = 6k + 9 = 3*(2k + 3)
=> Will be divisible by 3
Answer B

Hope it helps!