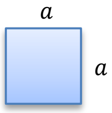
GMAT Geometry - LGOdream

GEOMETRY

Areas

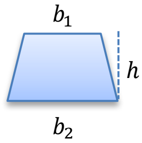
Square

$A = a^2$



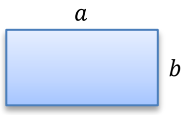
Trapezoid

$$A = \frac{h}{2}(b_1 + b_2)$$



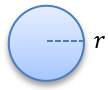
Rectangle

$A = ab$



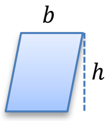
Circle

$A = \pi r^2$



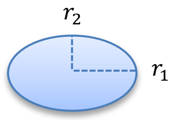
Parallelogram

$A = bh$



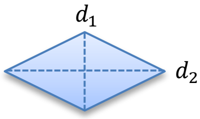
Ellipse

$A = \pi r_1 r_2$



Rhombus

$A = bh$

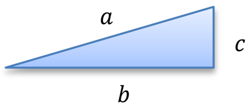


Max Area / Perimeter: of all the quadrilaterals, the square has the largest area and the minimum perimeter. For triangles, place two sides perpendicular to maximize the area.

Triangles & Polygons

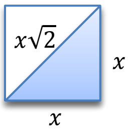
- Sum of the three angles of a triangle = 180.
- Sum of interior angles of a polygon $= (n-2)180$ ... OR ... divide the polygon into triangles.
- Angles correspond to their opposite sides. If twos sides are equal, their angles are also equal.
- The sum of any two sides is greater than the third side.
- Similar triangles with side lengths in ratio $a{:}\,b$ will have their areas in ratio $a^2{:}\,b^2$.

General rule: $a^2 = b^2 + c^2$



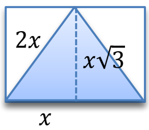
Isosceles triangle $(90-45-45)$

$a = x\sqrt{2}$
$b = x$
$c = x$



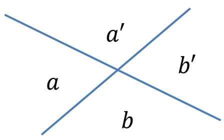
Equilateral triangle $(60-60-60)$

$a = 2x$
$b = x\sqrt{3}$
$c = x$

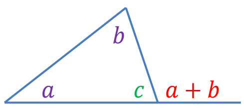


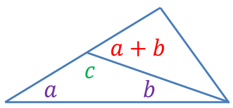
The above formulas apply only to rectangle triangles. If a triangle is not rectangle, divide it in rectangle triangles to solve.

Lines and angles



(a, b) and (a', b') are supplementary angles because they add up to 180°





Given any two angles of a triangle, the third angle's supplementary angle is equal to the sum of the first two angles.

Coordinate plane – Lines and points

Line formula

$$\text{General form: } Ax + By + C = 0$$

$$By = -Ax - C$$

$$y = -\frac{A}{B}x - \frac{C}{B}$$

$Slope - intercept\ form$: $\boldsymbol{y} = \boldsymbol{mx} + \boldsymbol{b}$

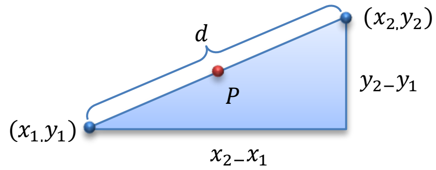
m: slope
b: y-intercept

Parallel lines (||) have the same slope: $-\frac{A_1}{B_1} = -\frac{A_2}{B_2}$

Perpendicular lines (⊥) have negative reciprocal slope $-\frac{A_1}{B_1} = \frac{B_2}{A_2} \rightarrow A_1A_2 - B_1B_2 = 0$

$Distance\ between\ two\ points\ (x_1, y_1)\ \&\ (x_2, y_2)$: $\boldsymbol{d} = \sqrt{(\boldsymbol{x_2} - \boldsymbol{x_1})^2 + (\boldsymbol{y_2} - \boldsymbol{y_1})^2}$

$Midpoint\ between\ two\ points\ (x_1, y_1)\ \&\ (x_2, y_2)$: $\boldsymbol{P} = \left(\frac{\boldsymbol{x_1} + \boldsymbol{x_2}}{\boldsymbol{2}}, \frac{\boldsymbol{y_1} + \boldsymbol{y_2}}{\boldsymbol{2}}\right)$



$Distance\ between\ a\ point\ (x_0, y_0)\ and\ a\ line$: $\boldsymbol{D} = \frac{|\boldsymbol{Ax_0} + \boldsymbol{Bx_0} + \boldsymbol{C}|}{\sqrt{\boldsymbol{A}^2 + \boldsymbol{B}^2}}$

$Slope\ of\ a\ line = \frac{\Delta y}{\Delta x} = \frac{\boldsymbol{y_1} - \boldsymbol{y_2}}{\boldsymbol{x_1} - \boldsymbol{x_2}}$

Find the equation of a line using two points of that line

1. Find slope $= m$
2. Plug one of the points (x, y) in the equation to find b.

Find two points of a line using the equation of that line [$\boldsymbol{slope} \neq \boldsymbol{0}$ ($\boldsymbol{perpendicular}$), $\boldsymbol{undefined}$ ($\boldsymbol{vertical}$)]:

1. Plug $x = 0$ to find the x-intercept point, $(0, y)$
2. Plug $y = 0$ to find the y-intercept point, $(x, 0)$

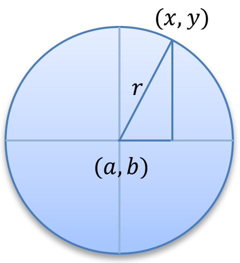
Coordinate Plane – Circles

Equation of a circle (applies to any point in the perimeter):

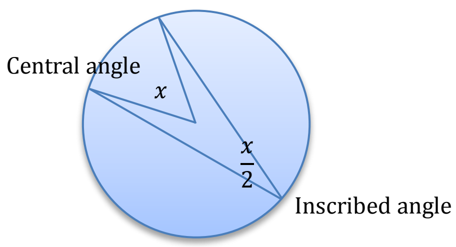
$(x - a)^2 + (y - b)^2 = r^2$

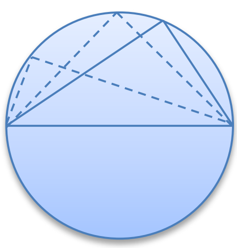
If the circle is centered at the origin $(a, b) = (0,0)$:

$x^2 + y^2 = r^2$



Other considerations





→
If one of the sides of an inscribed triangle is a Diameter of the circle, then the triangle must be rectangle.

Cylinder's area: top circle + bottom circle + rectangle.

Coordinate Plane – Parabolas

$\boldsymbol{y} = \boldsymbol{ax^2} + \boldsymbol{bx} + \boldsymbol{c}$

y–intercept: $\boldsymbol{c}$ $(\boldsymbol{x} = 0)$

x–intercept: solve the equation $\boldsymbol{x} = \frac{-\boldsymbol{b} \pm \sqrt{\boldsymbol{b}^2 - \boldsymbol{4ac}}}{\boldsymbol{2a}}$

$a \uparrow \Rightarrow width \downarrow$

$a > 0$

$a > 0$

$\sqrt{\boldsymbol{b}^2 - \boldsymbol{4ac}} > \boldsymbol{0}$ → 2 intercepts on x
$= \boldsymbol{0}$ → 1 intercept on x
$= \boldsymbol{0}$ → no intercepts on x

Vertex: $\left(-\frac{\boldsymbol{b}}{\boldsymbol{2a}}, \boldsymbol{y}\right)$ → find $\boldsymbol{x}$ then substitute and find $\boldsymbol{y}$