The formula Distance = Rate x Time expresses one of the most frequently used relations in algebra.

Since an equation remains true as long as you divide through by the same non-zero element on each side, this formula can be written in different ways:

- To find **rate**, divide through on both sides by *time:*
-                 Distance
-         Rate = -----------
-                   Time

  Rate is distance (given in units such as miles, feet, kilometers, meters, etc.) divided by time (hours, minutes, seconds, etc.). Rate can always be written as a fraction that has distance units in the numerator and time units in the denominator, e.g., 25 miles/1 hour.

- To find **time**, divide through on both sides by *rate:*
-                 Distance
-         Time = -----------
-                   Rate

When using this equation, it's important to keep the units straight. For instance, if the rate the problem gives is in miles per hour (mph), then the time needs to be in hours, and the distance in miles. If the time is given in minutes, you will need to divide by 60 to convert it to hours before you can use the equation to find the distance in miles. Always make your units match: if the time is given in fortnights and the distance in furlongs, then the rate should be given in furlongs per fortnight.

You can see why this is true if you look carefully at how the units are expressed. Say a car is travelling at 30 mph and you want to figure out how far it will go in 2 hours. You can use the formula:

```
    Rate    x  Time    = Distance

    miles
30 -------  x  2 hours = 60 miles
    hour
```

The hours cancel, leaving only miles.

What if you want to calculate the number of miles a car travelling 30 mph goes in 120 minutes?

Since 120 minutes is equal to two hours (60 minutes in one hour x 2 hours = 120 minutes), we should get the same distance of 60 miles, but we will *not* get the answer this way:

```
    miles                             mile minutes
30 ------- x 120 minutes = 3600 --------------
    hour                                  hour
```

Now, 3600 mile minutes per hour isn't very helpful, since we'd like our answer in miles. We need to divide by 60 minutes per hour:

```
      mile minutes       1 hour
3600 -------------- x ------------ = 60 miles
```

```
                hour         60 minutes
```

The hours and the minutes cancel, leaving only miles.

Although we can find an answer this way in the correct units, a better method would be to convert minutes to hours *before* using the formula.

Remembering to be careful about units, let's look at a problem.

Superheroes Liza and Tamar leave the same camp and run in opposite directions. Liza runs 1 mile per second (mps) and Tamar runs 2 mps. How far apart are they in miles after 1 hour?

To begin, we can either convert rates to miles per hour, or we can convert the time to seconds. Let's convert from miles per second to miles per hour.

There are 3600 seconds in an hour, so if Liza runs 1 mile in a second, then she will run at 3600 x 1 = 3600 mph. Similarly, Tamar will run at 3600 x 2 = 7200 mph.

```
   miles      3600 seconds         miles
2 -------- x -------------- = 7200 -------
   second       1 hour             hour
```

The seconds cancel, leaving miles per hour.

Back to the problem. How far does Liza run in one hour? We know her rate (3600 mph) and the time that she runs (one hour), so we can use the formula:

```
      miles
3600 ------- x 1 hour = 3600 miles
      hour
```

This makes sense because, by definition, if Liza's speed is 3600 miles per hour, then she runs 3600 miles in an hour.

Tamar, whose speed is 7200 miles per hour, will run 7200 miles in an hour.

How far apart will the two runners be after an hour? The answer is simply the sum of the distance each runs in an hour: 3600 + 7200 = 10,800 miles apart.

Since the earth has a circumference of about 24,000 miles at its equator, that's a little less than halfway around the world!

These problems, however, can be tricky:

Karen can row a boat 10 kilometers per hour in still water. In a river where the current is 5 kilometers per hour, it takes her 4 hours longer to row a given distance upstream than to travel the same distance downstream. Find how long it takes her to row upstream, how long to row downstream, and how many kilometers she rows.

One of the best ways to start a problem like this is to make a table that uses all the information you have been given. Let's make one for the information we have about the distance, rate, and time Karen travels when she is going both upstream and downstream. We'll call the time it takes to row downstream x, which means that the time it takes to row upstream is x +4.

We'll start by calculating Karen's rates going upstream and downstream. When she is traveling against the current, she won't be able to row 10 kilometers/hour. Her speed relative to the shore will only be 5 kilometers per hour because the force of the current, which is flowing at 5 kilometers/hour, slows her rate by 5 km/hour. When Karen is rowing downstream, however, the current helps her go faster, so she moves 10 + 5 = 15 km/hour.

We can use the formula, written as Rate x Time = Distance:

	Rate (km/hr)	Time (hr)	Distance (km)
Downstream	15	x	15x
Upstream	5	x+4	5(x+4)

Because Karen goes the same distance upstream and downstream, we know that the two expressions of distance - for upstream and downstream - must be equal; we can set the upstream distance equal to the downstream distance. This produces the following equation, which we solve for x:

```
Statement of original equation:  15x = 5(x+4)
Distributing on right side:      15x = 5x+20
Subtracting 5x from both sides:  10x = 20
Dividing both sides by 10:         x = 2
```

x equals the time it takes Karen to row downstream, or 2 hours. Since it takes her four hours longer to row upstream, this time will be 2 + 4 = 6 hours.

How many kilometers does she row? Look at the distance column in the table. Since x is in hours, Karen's downstream distance is 15 x 2 = 30 kilometers.

The problem states that Karen rows the same distance upstream as down. Let's check our work... yes, 5(2+4) = 5 x 6 = 30 kilometers.

As is frequently the case with word problems, setting up the equations is the hardest part. Once that's done, the rest is relatively easy. Remember always to answer what the question asks - don't stop once you've solve for x, because that may be only part of what the question asked - and *always* check your answer.

Distance, Rate and Time

```
Date: 3 Jun 1995 16:16:54 -0400
```

From: Richard Seguin
Subject: Question on Distance, Rate and Time.

I have a question like this:

Ian travelled by train at 80km/h and then by car at 90 km/h. It took him 3 h
to travel the total distance of 265km.

Ok, what I did was:

Train:

Speed = 80km/h
Time = X

(80)(x)=80x

Car:

Speed: 90km/h
Time 3-X

80x = 90 (3-x)
80x=270 - 90x
170x=270
x= 1.5

Train's Distance was 120 Km and it's time was 1.5H
Car's Distance was 145km and it's time was 1.5H

Is what I did right? Please reply ASAP! Before 8 EST would be appreciated.

Richard Seguin

Date: 6 Jun 1995 09:31:57 -0400
From: Dr. Ken
Subject: Re: Question on Distance, Rate and Time.

Hello there!

>Ian travelled by train at 80km/h and then by car at 90 km/h. It took him 3 h
>to travel the total distance of 265km.
. . .

>Car:
>
>   Speed: 90km/h
>   Time 3-X

I like this much. However, I think your next step isn't quite right. You say that the two distances 80x and 90(3-x) are equal, but I don't think we know that Ian travelled the same distance by each mode of transportation.

Instead, use the total distance 265 to make the equation

```
265 = 80x + 90(3-x)              since the two distances add to 265
265 = 80x + 270 - 90x
10x = 5
  x = .5
```

So it looks like Ian traveled 1/2 hour by train, and then 2.5 hours by car. That means he went 40 km by train, and the remaining 225 km by car. Sound good?

-K

Setting Up Proportions and Unit Conversions

Date: 7/25/96 at 20:36:31
From: Anonymous
Subject: Setting Up Proportions and Unit Conversions

If you can run 100 meters in 10 seconds, how long, in days, hours, and minutes, does it take you to run 12,800,000 meters?

Date: 7/26/96 at 0:38:13
From: Doctor Paul
Subject: Re: Setting Up Proportions and Unit Conversions

Let's set up a proportion:

```
100 meters      12,800,00 meters
---------- =    ----------------
10 seconds         x seconds
```

x = 1,280,000 seconds. Now we have to convert that to days:

```
                             1 hour          1 day
1,280,000 seconds    x    ------------  x   --------
                          3600 seconds      24 hours
```

so that's equal to 14.814 days. Let's now convert .814 days into hours:

```
.814 day       1 day
--------  =   -------
 x hours      24 hours
```

so .814 day = 19.536 hours

So right now we have 14 days, 19.536 hours. We need to convert .536 hours into minutes..

```
.536 hours       1 hour
---------- =     -------
 x minutes     60 minutes
```

x = 32.16 minutes.

Since your answer doesn't want seconds we round that to 32 minutes.

The answer is:
14 days, 19 hours, 32 minutes.

Running Problem (Time and Distance)

Date: 8/31/96 at 18:53:20
From: Anonymous
Subject: Time and distance problems

If Lucy runs 3 mph and Jan runs 7 mph, when will Jan catch up with Lucy if she gives Lucy a head start?

I think I solved it with a proportion, but I wonder if this is the best approach. Here is my "solution":

```
   3 mph : 7mph = x minutes : x + 10 minutes
3x + 30 minutes = 7x
     30 minutes = 4x
    7.5 minutes = x
   17.5 minutes = x + 10 minutes
```

Jan will catch up with Lucy 17.5 minutes after Lucy begins to run.

Date: 9/5/96 at 10:3:26
From: Doctor Jerry
Subject: Re: Time and distance problems

From your answer, the head start must be 10 minutes, or 1/6 hour. Right?

Solving problems by proportion is a very nice method, but you must be certain that the method fits. It doesn't fit here, because of the head start.

Imagine a number line, with both girls starting to run to the right from the 0 mark. At the time Jan starts, Lucy will be at the 3*(10/60) = 1/2 mile mark. Let a clock start running just as Lucy reaches the 1/2 mile mark. Let t be the clock time in hours. Lucy will be at the 1/2 + 3t mark and Jan will be at the 7t mark when the clock reads t hours.

To find out when Jan catches up, we look for the t for which

```
   1/2 + 3t = 7t.
```

I'm sure you can solve this equation for t. Remember that t is measured in hours.

Solving for Time

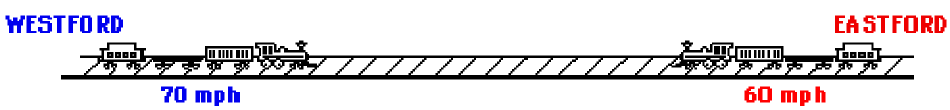
```
Date: 2/1/96 at 20:38:36
From: Anonymous
Subject: word problem

You are traveling 55 mph over a bridge that is 4260 ft. long. How
long does it take to cross the bridge?

Help me!
```

```
Date: 6/15/96 at 9:14:59
From: Doctor Robert
Subject: Re: word problem

Remember that distance = (rate)(time).  If you solve this for time
you get time = (distance)/(rate).

Your problem involves two different units of distance.  We must
convert miles to feet.  1 mile = 5280 ft.

The answer to your problem is then

time = 4260 ft/290400 ft/hr = .01467 hr

You can convert this to minutes by multiplying by 60.  The time
would be .8801 minutes or approximately 52.8 seconds.
```

The Two Trains

Two trains leave different cities heading toward each other at different speeds. When and where do they meet?

Train A, traveling 70 miles per hour (mph), leaves Westford heading toward Eastford, 260 miles away. At the same time Train B, traveling 60 mph, leaves Eastford heading toward Westford. When do the two trains meet? How far from each city do they meet?



To solve this problem, we'll use the distance formula:

Distance = Rate x Time

Since an equation remains true as long as we perform the same operation on both sides, we can divide both sides by rate:

$$\frac{\text{Distance}}{\text{Rate}} = \text{Time}$$

or by time:

$$\frac{\text{Distance}}{\text{Time}} = \text{Rate}$$

So rate is defined as distance divided by time, which is a ratio.

Speed is another word that is used for rate. When a problem says that a train is moving at a speed of 40 mph, you can understand this to mean that the train's rate is 40 mph, which means it will travel 40 miles in one hour.

Here are two different ways to approach this problem. Let's start by listing the information given:

Speed of Train A: 70 mph
Speed of Train B: 60 mph
Distance between Westford and Eastford: 260 miles

Method I: We'll use the notion of *relative speed* [1] (or relative rate) in order to express the rates of the two trains in one number that can then be used in the distance formula.

Imagine you're on Train A. You're going 70 mph, so your speed relative to the trees, houses, and other non-moving things outside the train is 70 mph. (All of those objects look as if they're going by at 70 mph.) Now imagine you're the engineer and you can see Train B coming toward you - not on the same track, of course! Since Train B is moving 60 mph, it will look as if it's approaching faster than if it were sitting still in the station - a lot faster than the trees and houses appear to be moving.

The relative speed of the two trains is the sum of the speeds they are traveling. (If you're on either of the trains, this is the speed you appear to be moving when you see the other train.) In our problem, the relative speed of the two trains is 70 mph + 60 mph = 130 mph. What if the trains were traveling in the same direction? Then we'd need to subtract the speed of the slower train from the speed of the faster train, and their relative speed would be 10 mph.

At this point we know two of the three unknowns: rate and distance, so we can solve the problem for time. Remember that time = distance/rate, the distance traveled is 260 miles, and the relative speed is 130 mph:

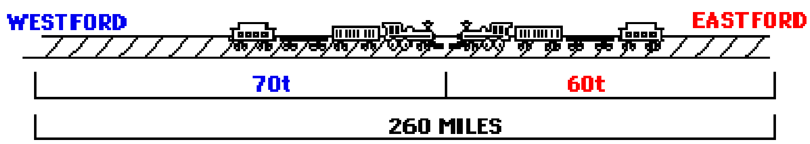
$$t = 260 \text{ miles}/130 \text{ mph}$$
$$t = 2 \text{ hrs.}$$

We find that **the trains meet two hours after leaving their respective cities**.

Method II: Here we'll begin by noting that the distance between Westford and Eastford is 260 miles: this is the total distance the trains will travel. Using the distance formula (Distance = rate x time, or D = rt) we can express the distance traveled by each train:

Train A moving at 70 mph in t hours will cover 70t miles
Train B moving at 60 mph in t hours will cover 60t miles

Together the two trains will cover the distance 70t + 60t



Since we know that this distance is 260 miles, we can write the following algebraic equation to represent this information.

$$70t + 60t = 260$$

Solving this equation we find that:

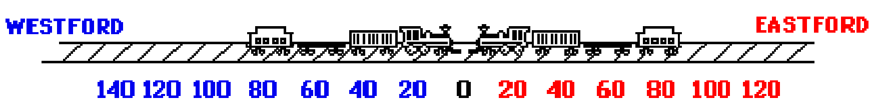
$$130t = 260$$
$$t = 2$$

which tells us that the trains will meet in 2 hours.

Now, **where do the trains meet?** We again use the distance formula to find how far each train has traveled in two hours:

For Train A: 70 mph x 2 hrs = 140 miles
For Train B: 60 mph x 2 hrs = 120 miles

Thus **the two trains meet at a point 140 miles from Westford and 120 miles from Eastford.**



To check this, we can add 140 to 120: the answer is 260, which was the given distance between the two cities.

Let's look at a variation on this problem.
Train A, traveling 40 mph, leaves Westford heading toward Eastford, 260 miles away. One hour after Train A leaves Westford, Train B, traveling 70 mph, leaves Eastford heading toward Westford. When do the two trains meet?

Notice that in this problem, the two trains do not leave their respective cities at the same time.

Method I: Let's move the starting point for Train A so we can treat the problem as if the trains leave at the same time, which we already know how to do.

We know that Train A is moving 40 mph, and will therefore travel 40 miles in the hour before Train B leaves Eastford. This means that by the time Train B starts moving, the two trains are only 260 - 40 = 220 miles apart. Now we can use the relative speed of the trains, which is 40 + 70 = 110 mph. Using the distance formula for time (time = distance/rate), we write:

$$t = 220 \text{ miles}/110 \text{ mph}$$
$$t = 2 \text{ hrs.}$$

Since t represents the time traveled by each train *after* Train A has already traveled for one hour, Train B travels 2 hours before meeting Train A. Adding the extra hour that Train A travels before Train B starts moving, we see that Train A must travel 3 hours before meeting Train B.

Method II: Let t represent the time that Train A travels. Since Train B leaves one hour after Train A, let t -1 represent the time that Train B travels.

Again, the sum of the distances traveled by the two trains up until the time they meet is 260 miles: between the two of them, they cover all 260 miles of track. Using the distance formula:

distance traveled by Train A = $40t$
distance traveled by Train B = $70(t-1)$

$$40t + 70(t-1) = 260 \text{ miles}$$
$$t = 3 \text{ hrs.}$$

Since t represents the time that Train A has been traveling, this means that Train A travels 3 hours before meeting Train B. But how long has Train B been traveling? Train B travels t -1 hours, which means that it travels 2 hours before meeting Train A.

Note:
To help us make sense of relative speed and how it relates to distance traveled, let's think of it in terms of the distance the two trains travel toward each other in one hour. We'll again use the distance formula:

Rate x Time = Distance

Train A's rate is 70 mph, so: 70 mph x 1 hr = 70 miles
Train B's rate is 60 mph, so: 60 mph x 1 hr = 60 miles

If we add these two numbers together, we get 130 miles, which means that 130 miles of track will be covered in one hour. This is the same as saying that the two trains are going 130 mph relative to each other, so:

130 mph x 1 hr = 130 miles

Either way, the numbers tell you that 130 miles of track will be covered in an hour.

What is a 'Work' Word Problem?

1) It involves a number of people or machines working together to complete a task.

2) We are usually given individual rates of completion.

3) We are asked to find out how long it would take if they work together.

Sounds simple enough doesn't it? Well it is!

There is just one simple concept you need to understand in order to solve any 'work' related word problem.

The 'Work' Problem Concept

STEP 1 : Calculate how much work each person/machine does in one unit of time (could be days, hours, minutes, etc).

How do we do this? Simple. If we are given that A completes a certain amount of work in X hours, simply reciprocate the number of hours to get the per hour work. Thus in one hour, A would complete (1/X) of the work.

But what is the logic behind this? Let me explain with the help of an example.

Assume we are given that Jack paints a wall in 5 hours.

This means that in every hour, he completes a fraction of the work so that at the end of 5 hours, the fraction of work he has completed will become 1 (that means he has completed the task).

Thus, if in **5** hours the fraction of work completed is **1**,

Then in **1** hour, the fraction of work completed will be **(1*1)/5**

STEP 2 : Add up the amount of work done by each person/machine in that one unit of time.

This would give us the total amount of work completed by both of them in one hour. For example, if **A completes (1/X)** of the work in one hour and **B completes (1/Y)** of the work in one hour, then **TOGETHER**, they can complete **[(1/X) + (1/Y)]** of the work **in one hour**.

STEP 3 : Calculate total amount of time taken for work to be completed when all persons/machines are working together.

The logic is similar to one we used in STEP 1, the only difference being that we use it in reverse order. Suppose [(1/X) + (1/Y)] = (1/Z). This means that **in one hour**, A and B **working together** will complete **(1/Z)** of the work. **Therefore, working together, they will complete the work in Z hours.**

My advice here would be : DON'T go about these problems trying to remember some formula. Once you understand the logic underlying the above steps, you will have all the information you need to solve any 'work' related word problem. (You will see that the formula you might have come across can be very easily and logically deduced from this concept).

Now, lets go through a few problems so that the above-mentioned concept becomes crystal clear.

Lets start off with a simple one :

Example 1.

Jack can paint a wall in 3 hours. John can do the same job in 5 hours. How long will it take if they work together?

Solution 1.

This is a simple straightforward question wherein we must just follow steps 1 to 3 in order to obtain the answer.

STEP 1 : Calculate how much work each person does in one hour.
Jack → (1/3) of the work
John → (1/5) of the work

STEP 2 : Add up the amount of work done by each person in one hour.
Work done in one hour when both are working together = (1/3)+(1/5) = (8/15)

STEP 3 : Calculate total amount of time taken when both work together.
If they complete **(8/15)** of the work in **1** hour,
then they would complete **(1)** job in **(1*1)/(8/15)** hours.

Therefore answer is (15/8) hours.

Simple, wasn't it? Now lets move onto one that is slightly trickier.

Note: As we move on to trickier problems, we will refrain from using any specific approach (Eg. Step 1, Step 2, Step 3, etc.) since that might serve only to confuse us when the problem involves a lot of twists and turns. Armed with a clear understanding of the concept, we should be able to tackle any 'work' related word problem, no matter how convoluted, by applying our knowledge to the information at hand.

Example 2.

Working, independently X takes 12 hours to finish a certain work. He finishes 2/3 of the work. The rest of the work is finished by Y whose rate is (1/10) of X. In how much time does Y finish his work?

Solution 2.

Now the only reason this is trickier than the first problem is because the sequence of events are slightly more complicated. The concept however is the same. So if our understanding of the concept is clear, we should have no trouble at all dealing with this.

'Working, independently X takes 12 hours to finish a certain work'
This statement tells us that in one hour, X will finish (1/12) of the work.

'He finishes 2/3 of the work'
This tells us that (1/3) of the work still remains.

'The rest of the work is finished by Y whose rate is (1/10) of X'
Y has to complete (1/3) of the work.
His rate is (1/10) that of X.
We have already calculated rate at which X works to be (1/12).
Therefore, rate at which Y works is (1/10)*(1/12) = (1/120)

'In how much time does Y finish his work?'
If Y completes **(1/120)** of the work in **1** hour,

Then he will complete **(1/3)** of the work in **[(1/3)*1]/(1/120)** = 40 hours.

Therefore answer is 40 hours.

So as you can see, even though the question might have been a little difficult to follow at first reading, the solution was in fact quite simple. We didn't use any new concepts. All we did was apply our knowledge of the concept we learnt earlier to the information in the question in order to answer what was being asked.

Example 3.

Working together, printer A and printer B would finish a task in 24 minutes. Printer A alone would finish the task in 60 minutes. How many pages does the task contain if printer B prints 5 pages a minute more than printer A?

Solution 3.

This problem is interesting because it tests not only our knowledge of the concept of word problems, but also our ability to 'translate English to Math'

'Working together, printer A and printer B would finish a task in 24 minutes'
This tells us that A and B combined would work at the rate of (1/24) per minute.

'Printer A alone would finish the task in 60 minutes'
This tells us that A works at a rate of (1/60) per minute.

At this point, it should strike you that with just this much information, it is possible to calculate the rate at which B works.
Rate at which B works = (1/24) – (1/60) = (1/40)

'B prints 5 pages a minute more than printer A'
This means that the difference between the amount of work B and A complete in one minute corresponds to 5 pages.
So let us calculate that difference. It will be = (1/40) – (1/60) = (1/120)

'How many pages does the task contain?'
If **(1/120)** of the job consists of **5** pages,
Then the **1** job will consist of **(5*1)/(1/120)** = 600 pages

Therefore answer is 600 pages.

Hopefully you're getting the hang of it by now.

Lets move on to a really tough one.

Example 4.

Machine A and Machine B are used to manufacture 660 sprockets. It takes machine A ten hours longer to produce 660 sprockets than machine B. Machine B produces 10% more sprockets per hour than machine A. How many sprockets per hour does machine A produce?

Solution 4.

You wont come across many problems tougher than this on the GMAT. But as we will see, even the toughest of problems can be solved with relative ease if we employ the concept discussed above.

'It takes machine A ten hours longer to produce 660 sprockets than machine B'
Let machine A produce 660 sprockets in 'x' hours.
Therefore, machine B will produce 660 sprockets in 'x – 10' hours.

With this information, we can calculate the amount of work machine A and B do per hour respectively.
Rate at which machine A works = [1/x] per hour
Rate at which machine B works = [1/(x – 10)] per hour

'Machine B produces 10% more sprockets per hour than machine A'
If machine A produces [1/x] sprockets an hour, then machine B will produce (1/x) +(10/100)*(1/x) = (11/10x)

But we already know that rate at which machine B works = [1/(x – 10)] per hour. Therefore, equating it to (11/10x) we get the following equation:

(11/10x) = 1/(x-10) → x = 110 hours

'How many sprockets per hour does machine A produce?'
If in **110** hours A produces **660** sprockets,
Then in **1** hour it will produce **(660*1)/110** = 6

Therefore answer is 6.

As you can see, the main reason the 'tough' problems are 'tough' is because they test a number of other concepts apart from just the 'work' concept. However, once you manage to form the equations, they are really not all that tough.

And as far as the concept of 'work' word problems is concerned – it is always the same!